Effect of Speed Control Humps on Vehicle Dynamics

by

Yaagoub Nassar Al-Nassar

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

MECHANICAL ENGINEERING

February, 1981

بسم الله الرحمن الرحيم

EFFECT OF SPEED CONTROL HUMPS ON VEHICLE DYNAMICS

by

YAAGOUB NASSAR AL-NASSAR

A Thesis Presented to the

COLLEGE OF GRADUATE STUDIES

UNIVERSITY OF PETROLEUM AND MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirement for the Degree in

MASTER OF SCIENCE

in

MECHANICAL ENGINEERING

February 1981

UMI Number: 1381121

UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

This thesis, written by

YAAGOUB N. AL-NASSAR

under the direction of his Thesis Committee, and approved by its members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING.

Dean, College of Graduate Studies

Date:

Department Chairman

THESIS COMMITTEE

Chairman (Dr. Essam Kassem)

Anwar Khalil Sheikh
Member (Dr. Anwar K. Sheikh)

Member (Dr. Abdulla I. Sayyari)

ABSTRACT

Speed control humps are effective in controlling speeding vehicles in residential areas, school exits, and university grounds. They should be well designed, properly constructed and supported by efficient warning signs.

The objective of this study is to investigate the effects of hump parameters (cross section, dimensions, spacing, etc.,) on the dynamic behaviour of the vehicle and the driver. Vehicle safety and driver comfort are most important in evaluating the effectiveness of speed control humps.

A mathematical model representing the vehicle and the driver is developed. Several hump profiles are tried. A "basic" hump is selected and the effects of the individual parameters of the hump on the dynamic response of the system are evaluated through the computer simulation. The results are analysed, conclusions and recommendations for the design of speed control humps are specified.

ACKNOWLEDGEMENTS

I wish to express my deep appreciation to Dr. Essam H. Kassem for his guidance and encouragement throughout this research. His broad background and experience and his unique ability to recognize the important have allowed him to identify and support this work. I am indebted to him and to other members of my Thesis Committee, Dr. Anwar K. Sheikh, and Dr. Abdulla I. Sayyari.

TABLE OF CONTENTS

Topic	Page
1 INTRODUCTION	1
1.1 General background	1
1.2 Hump parameters	6
1.3 Objectives	7
2 MATHEMATICAL MODELING OF VEHICLE AND DRIVER	9
2.1 Introduction	9
2.2 The Model	10
2.3 Vehicle and Driver performance	17
2.4 Computer program	17
3 SELECTION OF A HUMP	19
3.1 Introduction	19
3.2 Speed effects on hump dynamics	19
4 CASE STUDY	25
4.1 Introduction	25
4.2 CASE (I): Effect of smoothing the inlet and outlet of the hump	27
4.3 CASE (II): Effect of varying the width of the hump	32
4.4 CASE (III): Effect of varying the maximum height of the hump	36
4.5 CASE (IV): Effect of adding a flat top to the hump	40
4.6 CASE (V): Effect of vehicle crossing speed	45
4.7 CASE (VI): Effect of crossing the hump with deceleration	49
4.8 CASE (VII): Double hump	53
4.9 CASE (VIII): Triple hump	57
5 CONCLUSIONS AND RECOMMENDATIONS	61
REFERENCES	63
APPENDIX A: COMPUTER PROGRAM	65

List of Figures

Fig. No.	*Title*	*Page*
1	Typical cross section and dimensions of humps	3
2	Cross section of a hump	4
3	Cross section and hump dimensions	5
4	Model of the vehicle and driver	11
5	Cycloidal hump profile	13
6	Harmonic hump profile	14
7	Parabolic hump profile	15
8	Circular hump profile	16
9	Dynamic effects of an ideal hump	20
10	Effect of crossing speed on the acceleration of the driver	21
11	Effect of crossing speed on the vertical displacement of the vehicle's center of mass	22
12	Effect of crossing speed on the pitching of the vehicle	23
13	Harmonic hump with smooth inlet and outlet	28
14	Effect of smoothing the hump on the acceleration of the driver.	29
15	Effect of smoothing the hump on the displacement of the driver	29
16	Effect of smoothing the hump on the heaving of the vehicle	30
17	Effect of smoothing the hump on the motion of the front wheels	30
18	Effect of smoothing the hump on the motion of the rear wheels	1
19	Effect of smoothing the hump on the pitching of the vehicle	31
20	Effect of increasing the width of the hump on the acceleration of the driver	33

Fig. No.	*Title*	*Page*
21	Effect of increasing the width of the hump on the displacement of the driver	33
22	Effect of increasing the width of the hump on the heaving of the vehicle	34
23	Effect of increasing the width of the hump on the motion of the front wheels	34
24	Effect of increasing the width of the hump on the motion of the rear wheels	35
25	Effect of increasing the width of the hump on the pitching of the vehicle	35
26	Effect of increasing the height of the hump on the acceleration of the driver	37
27	Effect of increasing the height of the hump on the displacement of the driver	37
28	Effect of increasing the height of the hump on the heaving of the vehicle	38
29	Effect of increasing the height of the hump on the motion of the front wheels	38
30	Effect of increasing the height of the hump on the motion of the rear wheels	39
31	Effect of increasing the height of the hump on the pitching of the vehicle	39
32	Effect of wide humps on the acceleration of the driver	42
33	Effect of wide humps on the displacement of the driver	42
34	Effect of wide humps on the heaving of the vehicle	43
35	Effect of wide humps on the motion of the front vehicle	43
36	Effect of wide humps on the motion of the rear wheels	44
37	Effect of wide humps on the pitching of the vehicle	44
38	Effect of crossing speed on the acceleration of the driver	46
39	Effect of crossing speed on the displacement of the driver	46

Fig. No.	Title	Page
40	Effect of crossing speed on the heaving of the vehicle	47
41	Effect of crossing speed on the motion of the front wheels	47
42	Effect of crossing speed on the motion of the rear wheels	48
43	Effect of crossing speed on the pitching of the vehicle	48
44	Effect of deceleration on the acceleration of the driver	50
45	Effect of deceleration on the displacement of the driver	50
46	Effect of deceleration on the heaving of the vehicle	51
47	Effect of deceleration on the motion of the front wheels	51
48	Effect of deceleration on the motion of the rear wheels	52
49	Effect of deceleration on the pitching of the vehicle	52
50	Effect of the double hump on the acceleration of the driver	54
51	Effect of the double hump on the displacement of the driver	54
52	Effect of the double hump on the heaving of the vehicle	55
53	Effect of the double hump on the motion of the front wheels	55
54	Effect of the double hump on the motion of the rear wheels	56
55	Effect of the double hump on the pitching of the vehicle	56
56	Effect of the triple hump on the acceleration of the driver	58
57	Effect of the triple hump on the displacement of the driver	58

Fig. No.	Title	Page
58	Effect of the triple hump on the heaving of the vehicle	59
59	Effect of the triple hump on the motion of the front wheels	59
60	Effect of the triple hump on the motion of the rear wheels	60
61	Effect of the triple hump on the pitching of the vehicle	60

NOMENCLATURE

a	Distance between driver's seat and vehicle center of mass, m
B	Wheel base, m
C_f	Damping coefficient for front suspension, N.sec/m
C_r	Damping coefficient for rear suspension, N.sec/m
C_s	Damping coefficient for driver's seat, N.sec/m
C_t	Damping coefficient for tires, N.sec/m
h	Hump elevation, m
H	Hump maximum height, m
H_f	Input disturbance to front wheels, m
H_r	Input disturbance to rear wheels, m
I	Vehicle moment of inertia, kg.m^2
K_f	Stiffness of front suspension, N/m
K_r	Stiffness of rear suspension, N/m
K_s	Stiffness of driver's seat, N/m
K_t	Stiffness of tire, N/m
l_1	Distance between front wheels and center of mass, m
l_2	Distance between rear wheels and center of mass, m
M_s	Mass of the driver, kg
M_t	Mass of the wheel, kg
M_v	Mass of the vehicle, kg
S	Half the width of the hump, m
t	Time, secs
V	Vehicle speed, m/s
X_f	Displacement of front wheels, m
X_r	Displacement of rear wheels, m

X_s Displacement of driver's seat, m

X_v Displacement of vehicle center of mass, m

θ Rotation of the vehicle, rad.

CHAPTER 1

INTRODUCTION

1.1 GENERAL BACKGROUND

The need often arises to discourage motorists from travelling at high speeds along certain roads or using routes which are not suitable for through traffic as short cuts. Of the various ways of achieving this, one possibility is the construction of humps or undulations across the road. Speed control humps have become increasingly common on private access roads in areas such as the grounds of universities and camping sites.

The ideal speed control hump should be made such that at and below the design speed, all drivers should be able to cross the hump without damage to load or vehicle, or loss of control and they should suffer no discomfort. Above the design speed, the driver should suffer a degree of discomfort depending on the amount by which he violates the design speed, but there should still be no damage to load or vehicle or risk of loss of control. Unfortunately, the type of hump commonly used is often short, high and of rather severe profile. Such humps may damage the bottom of vehicles.

A comprehensive study of speed control humps was carried out by the Transport and Road Research Laboratory, England, 1973 [13]. Several humps of a segment of a circle in section were investigated. Seven vehicles including private cars, goods vehicles, a moped and a bus were used in the tests. Six subjects made estimates of the "noticeability" and "discomfort" of the different humps at various crossing speeds. Subjects recorded how uncomfortable the ride over the hump was after crossing at a specific speed. A seven-point rating scale was used to quantitatively describe the discomfortability. In a similar way, subjects also reported how noticeable the hump was, taking into account any noise or vibration that was experienced. Vertical acceleration experienced by subjects is the main factor influencing ride comfort. An accelerometer was used to record the vertical acceleration of the subject. High speed cine film was taken of the vehicles as it crossed various humps. The maximum displacements of the centers of the wheels and the maximum angular and vertical displacements of the car body were calculated. The results of the tests indicated that short humps tend to have characteristics which are the converse of those desired, i.e., their effects tend to become less prominent at the higher approach speeds. The study concludes that a hump 3.7 m wide and 100 mm high is most suitable for use in residential areas. Typical cross section and dimensions of humps used in the tests are shown in Fig.1. This hump produced an uncomfortable ride in most of the vehicles tested at speeds in excess of 32 km/hr. At the low speed of 8 km/hr, drivers of all vehicles could cross the hump with reasonable comfort. Humps of the above mentioned cross section were installed on an experi-

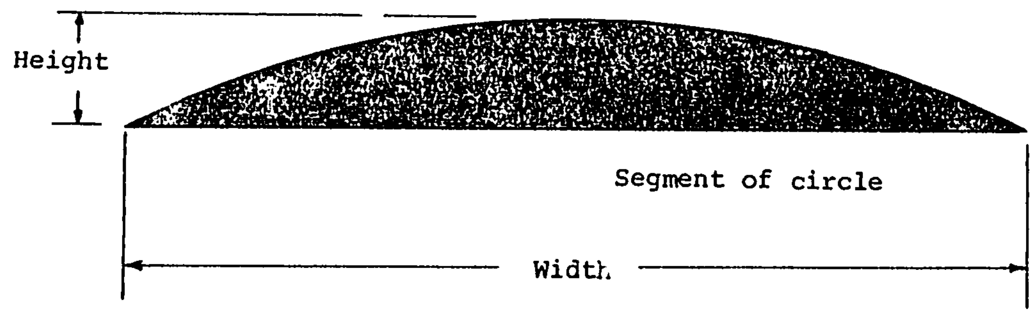


Fig.1 Typical cross section and dimensions of humps, [13]

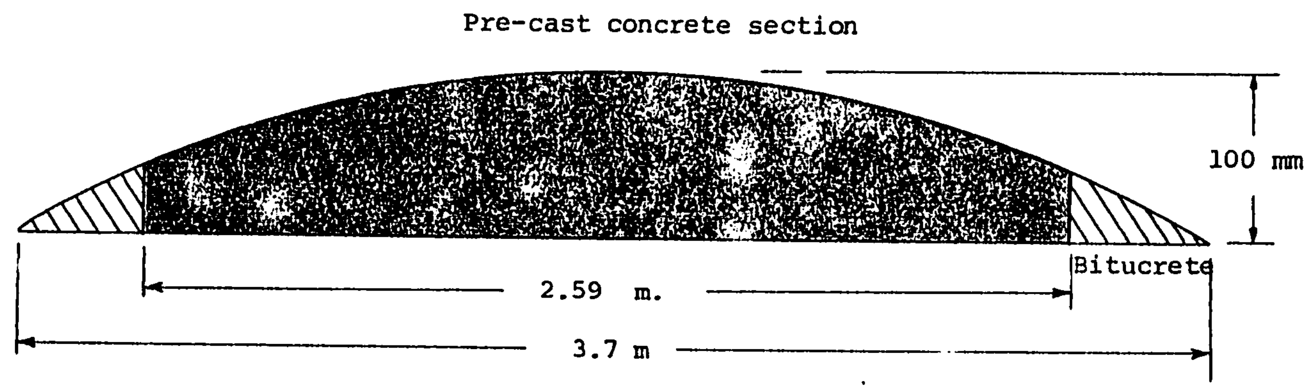


Fig.2 Cross Section of a Hump, [12]

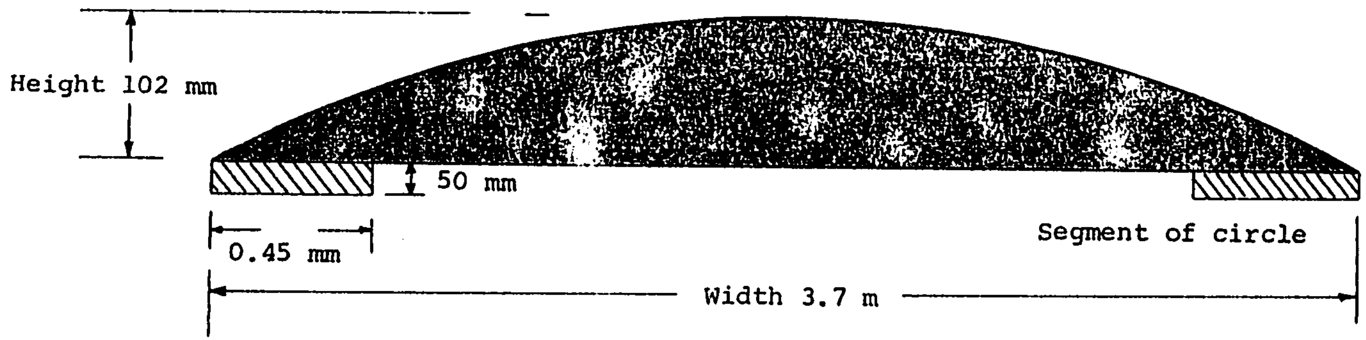


Fig.3 Cross Section and Hump Dimensions, [11]

mental basis in Cuddesdon Way, Cowley, Oxford, [12]. The humps were constructed from pre-cast concrete sections 0.6 m long and 2.59 m wide, the remainder of the profile being filled in with Bitucrete, Fig.2. That design was intended to facilitate removal. The results of another application of speed control humps is reported by Sumner and Baguley, [11]. Details are given of the method of construction, (see Fig.3), public reaction and effects on traffic and accidents.

1.2 HUMP PARAMETERS

Dimensions and configuration of the hump play a major role in its dynamic effects on vehicle and driver. Of particular importance are the following parameters:

a : HUMP PROFILE

The hump profile refers to the geometry of the cross section of the hump. Such profile can be circular, (i.e., a segment of a circle), parabolic, cycliodal, harmonic or connected segments of straight lines. Selection of a specific profile depends on its dynamic effects on vehicle and driver (which is the subject of this study), means of construction, practicality of the design, cost and materials used.

b : HEIGHT OF HUMP

Low humps are ineffective. High humps can be very dangerous and they might cause damage to the vehicle underneath. The maximum height of the hump is, in any case, limited by the ground clearance of low slung vehicles. The main factor in choosing a suitable height for the hump is the dynamic response of the vehicle.

c : WIDTH OF HUMP

There are two basic classes of humps, those which are short enough to be straddled by the wheels of all normal vehicles, and long humps which cannot be straddled except by a minority of large vehicles. It is known that short humps may be crossed at high speeds without undue discomfort to the vehicle occupants. Long humps by their nature provide a less ramp effect and a longer crossing time. The wide hump is created by adding a flat top to the short hump. A greater height, however, may be used without fear of grounding low slung vehicles.

d : MULTIPLE HUMPS

Humps can be used in groups. If the spacing between humps is relatively short (in the order of hump's width), then the purpose of such design is to magnify the dynamic effects on vehicle. On the other hand, if the spacing is long, then the purpose of such system is to control vehicle speeds over a long distance of the road. The vehicle can only reach a safe maximum speed between the humps.

1.3 OBJECTIVES

The objective of this study is to investigate the effect of hump profile, height, width and multiple humps on vehicle parameters (dimensions, inertia, speed, ..., etc.) which the result of specific dynamic response. A mathematical model is used to simulate vehicle performance and driver response under road humps. Different hump profiles are used. The effect of smoothing the inlets and outlets of humps is investigated. Hump dimensions (height, width) are varied to study their significance. Double and triple humps are used to evaluate vehicle performance under multiple humps. The vehicle response in terms of its pitch and heave motions and dynamic forces on the suspension system are examined. Ride comfortability in terms of the acceleration of the driver is evaluated.

CHAPTER 2

MATHEMATICAL MODELING OF VEHICLE AND DRIVER

2.1 INTRODUCTION

A mathematical model is required to study the dynamic effects of speed control humps on vehicle and driver. An optimal model that suits the study of whole-body vibration of road vehicles is investigated by Dahlberg [1]. The model is a linear, one-degree-of-freedom system. It is subjected to stationary zero mean Gaussian random excitation. A similar model is used by Sachs to study the effect of an adaptive suspension system in heavy commercial and military vehicles, [8]. In another application, Dahlberg modeled the vehicle as a linear two-degree-of-freedom system. The two-mass system is used to design an optimal speed-controlled suspension of a vehicle travelling on a randomly profiled road, [2]. Another two-degree-of-freedom system is used by Karnopp, [5]. The two degrees of freedom in this case, however, are the pitching and heaving motions of the vehicle mass. The model is used to study the dynamic response of vehicles traversing irregular surfaces. A four-degree-of-freedom model is formulated to study the heave-pitch motion of vehicles crossing elevated, flexible, randomly irregular spans. The vehicle model is supported by identical front and rear suspensions which consist of a secondary spring, a damper, an unsprung mass and a primary spring, [10]. A rather complex mathematical model has been set up by Rinonapoli and Bergomi, [7]. The model has 14 degrees of freedom and treats the vehicle as an assembly of rigid masses connected together and to the road by flexible elements. The model is used to predict car handling behavior on smooth and bumpy roads.

2.2 THE MODEL

The vehicle model chosen for this study is a five-degree-of-freedom one. The model is shown in Fig.4, and is similar to the one described in Ref.[3]. The model consists of a sprung mass (2000 kg), front suspension of stiffness (40000 N/m) and damping (3000 N.s/m), rear suspension of stiffness (40000 N/m) and damping (6000 N.s/m), two front and rear unsprung masses (50 kg) supported by elastic tires of stiffness (180000 N/m) and damping (350 N.s/m). The driver is simulated by a 100 kg mass and supported by a seat of stiffness (10000 N/m) and damping (500 N.s/m). The vehicle has a mass moment of inertia of 2000 kg m^2 in pitching motion. The center of mass is assumed to be 1.3 m from the front wheels and 1.2 m from rear ones, i.e., the wheel base is equal to 2.5 m. The center of mass of the driver is assumed 0.4 m from the center of mass of vehicle. These data are compatable with those given in Ref.[6, 4].

The front and rear wheel vertical displacements make up two of the degrees of freedom. Another two are the wholebody vertical displacement and rotation (the pitch movement) measured at the mass center.

The fifth degree of freedom enters into the system.

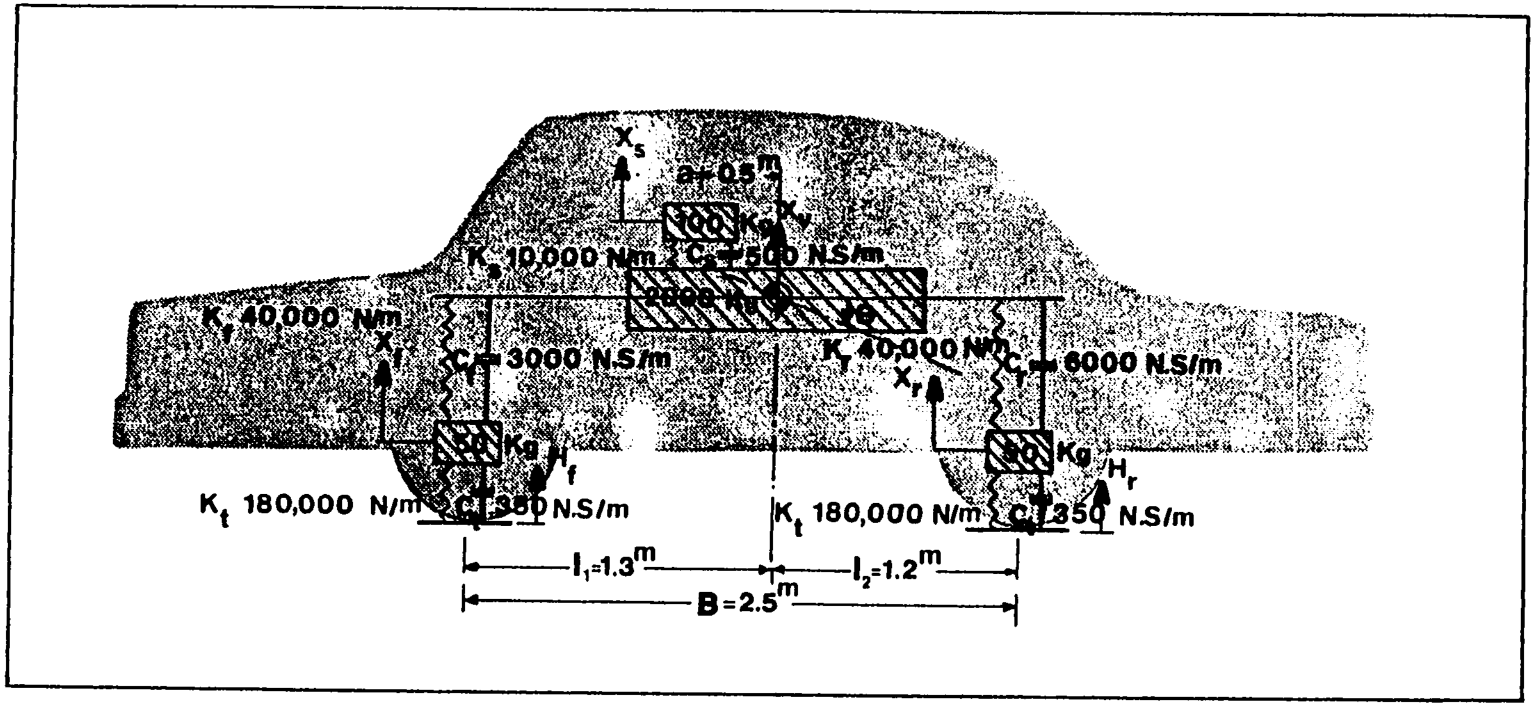


Fig.4 Model of the vehicle and driver

as the vertical displacement of the driver. The model is excited through the vertical displacements of the front and rear wheels as the vehicle crosses the hump.

Mathematically the model can be expressed in the following form:

$$M_s \ddot{X}_s + K_s[X_s - (X_v + a\theta)] + C_s[(\dot{X}_s - (\dot{X}_v + a\dot{\theta})] = 0 \quad (1)$$

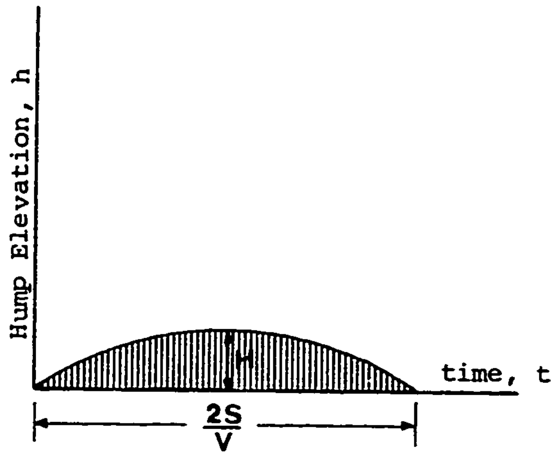
$$M_v \ddot{X}_v + K_s[X_v + a\theta) - X_s] + C_s[(\dot{X}_v + a\dot{\theta}) - \dot{X}_s] + K_f[X_v + \ell_1\theta - X_f]$$
$$C_f[\dot{X}_v + \ell_1\dot{\theta} - \dot{X}_f] + K_r[X_v - \ell_2\theta - X_r] + C_r[\dot{X}_v - \ell_2\dot{\theta} - \dot{X}_r] = 0 \quad (2)$$

$$M_t \ddot{X}_f + K_f[X_f - (X_v + \ell_1\theta)] + C_f[\dot{X}_f - (\dot{X}_v + \ell_1\dot{\theta})] + K_t[X_f] + C_t[\dot{X}_f]$$
$$= K_t H_f + C_t \dot{H}_f \quad (3)$$

$$M_t \ddot{X}_r + K_r[X_r - (X_v - \ell_2\theta)] + C_r[\dot{X}_r - (\dot{X}_v - \ell_2\dot{\theta})] + K_t[X_r] + C_t[\dot{X}_r]$$
$$= K_t H_r + C_t \dot{H}_r \quad (4)$$

$$I_v \ddot{\theta} + K_f \ell_1[(X_v + \ell_1\theta) - X_f] + C_f \ell_1[\dot{X}_v + \ell_1\dot{\theta} - \dot{X}_f]$$
$$- K_r \ell_2[(X_v - \ell_2\theta) - X_r] - C_r \ell_2[\dot{X}_v - \ell_2\dot{\theta} - \dot{X}_r]$$
$$+ K_s a[(X_v + a\theta) - X_s] + C_s a[(\dot{X}_v + a\dot{\theta}) - \dot{X}_s] = 0.0 \quad (5)$$

Four different hump profiles are used: cycloidal, harmonic, circular, and parabolic. The profiles alongwith their mathematical expressions as the vehicle feels them are shown in Figs.5 through 8. These profiles can be classified as "sharp profiles", i.e., the slopes at the inlet and outlet of the profile are not zero. The size of the hump can be changed through varying its width and height. Also a wide hump can be generated through adding

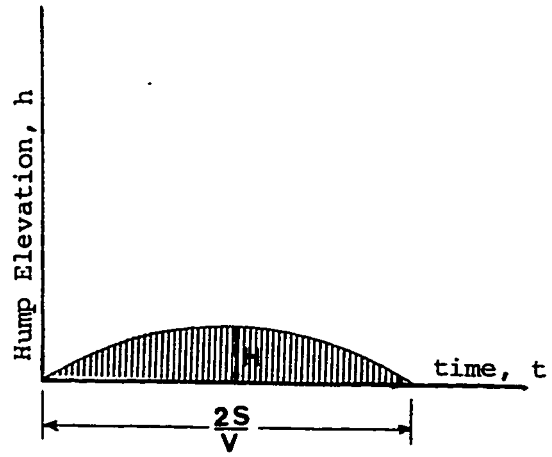


$$h = H\left[\frac{V.t}{S} + \frac{1}{\pi}\,\mathrm{Sin}\left(\pi\cdot\frac{V.t}{S}\right)\right] \qquad 0 \leqslant t \leqslant \frac{S}{V}$$

$$h = H\left[1\cdot - \frac{V.(t - S/V)}{S} + \frac{1}{\pi}\,\mathrm{Sin}\left(\pi\cdot\frac{V.(t - S/V)}{S}\right)\right] \qquad \frac{S}{V} \leqslant t \leqslant \frac{2S}{V}$$

where V = Vehicle speed

Fig.5 Cycloidal Hump Profile

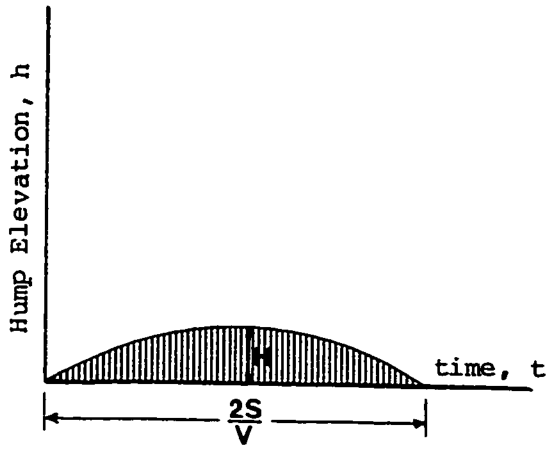


$$h = H\left[\mathrm{Sin}\left(\pi \cdot \frac{v.t}{2S}\right)\right] \qquad 0 \leqslant t \leqslant \frac{S}{v}$$

$$h = H\left[\mathrm{Cos}\left(\pi \cdot \frac{v\ (t - S/v)}{2S}\right)\right] \qquad \frac{S}{v} \leqslant t \leqslant \frac{2S}{v}$$

where v = Vehicle speed

Fig.6 Harmonic Hump Profile

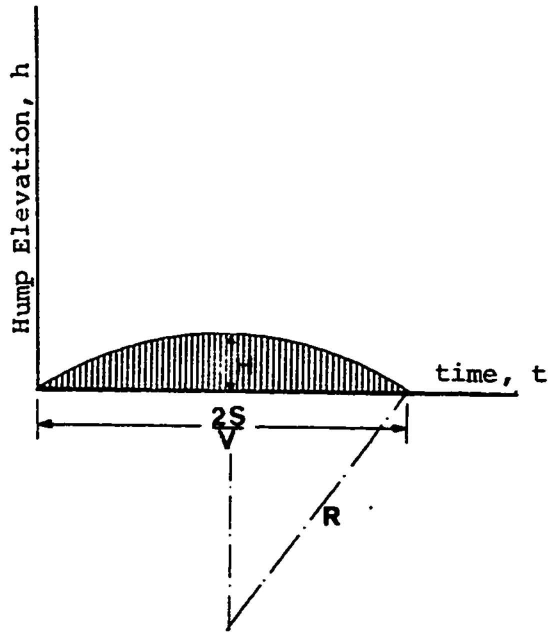


$$h = \frac{H.V.t}{S}\left[2. - \frac{V.t}{S}\right] \qquad 0 \leqslant t \leqslant \frac{S}{V}$$

$$h = H\left[1. - \left\{\frac{V\ (t - S/V)}{S}\right\}^2\right] \qquad \frac{S}{V} \leqslant t \leqslant \frac{2S}{V}$$

where V = Vehicle speed

Fig.7 Parabolic Hump Profile



$$h = \sqrt{R^2 - (t \cdot V - S)^2} - (R - H) \qquad 0 \leq t \leq \frac{2S}{V}$$

where V = Vehicle speed

Fig.8 Circular Hump Profile

a flat middle portion to the hump.

2.3 VEHICLE AND DRIVER PERFORMANCE

Vehicle and driver response to the humps are evaluated in terms of the following measures:

1 Driver's vertical acceleration, $\ddot{X}_s$, is a good measure of riding comfort, [9]. It is directly proportional to the inertia forces that the driver feels while crossing the hump.

2 Relative dispalcement of the driver with respect to the vehicle ($X_s - X_v$) is an important parameter in insuring that the driver's head will not hit the top of the vehicle.

3 Vertical and rotational accelerations of the vehicle ($\ddot{X}_v$ and $\ddot{\theta}$) are required to calculate inertia forces and moments acting of the chasis.

4 Vertical and rotation motions of the chasis (X_v and θ) are indications of the possibility of the vehicle damage if it hits the ground.

5 Displacements of front and rear wheels (X_f and X_r) are directly related to safety. Under some severe cases, the wheels can be deflected by a large amount causing the suspension to hit the "bump stops" producing a jolt. In some cases, the wheels can leave the ground which results in loss of contact with the pavement.

2.4 COMPUTER PROGRAM

Runge Kutta numerical method is used to solve the system of equations 1-5. This method requires the system of equations to be of the first order. Each of the equations is replaced by two first order equations. The ten initial conditions required are the initial positions and velocities: X_s, X_v, X_f, X_r, θ, $\dot{X}_s$, $\dot{X}_v$, $\dot{X}_f$, $\dot{X}_r$ and $\dot{\theta}$. The input function is the hump profile and its rate of change as felt at front and rear wheels: H_f, $\dot{H}_f$, H_r and $\dot{H}_r$.

Appendix (A) lists the program used to solve the developed system of equations.

CHAPTER 3

SELECTION OF A HUMP

3.1 INTRODUCTION

As it was mentioned in Chapter (1) the ideal speed control hump should be made such that at and below the design speed, all drivers should be able to cross the hump without damage to vehicle, or loss of control and they should suffer no discomfort. Above the design speed, the driver should suffer a degree of discomfort depending on the amount by which he violates the design speed, but there should still be no damage to load or vehicle or risk of loss of control. Using engineering terms, an ideal hump is the one which provides dynamic effects that are linearly proportional to vehicle speed. This is represented by the straight line shown in Fig.9. The hump which shows decreasing dynamic effects with speed is not good since it can be crossed with less discomfort at high speeds. On the other hand, the hump which shows increasing dynamic effects is also not acceptable since it over-penalizes the speeding driver.

3.2 SPEED EFFECTS ON HUMP DYNAMICS

Unfortunately, there is no profile that can produce such ideal effects. To demonstrate that concept, the dynamic effects of the four selected profiles (cycloidal, circular, sparabolic, and harmonic) are evaluated. They are all equal in width (1.5 m) and height (0.15 m). Three

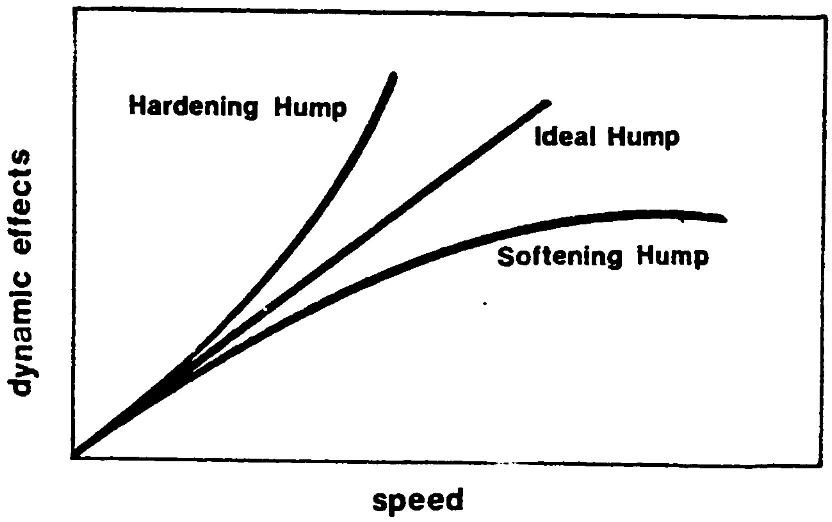


Fig.9 Dynamic effects of an ideal hump

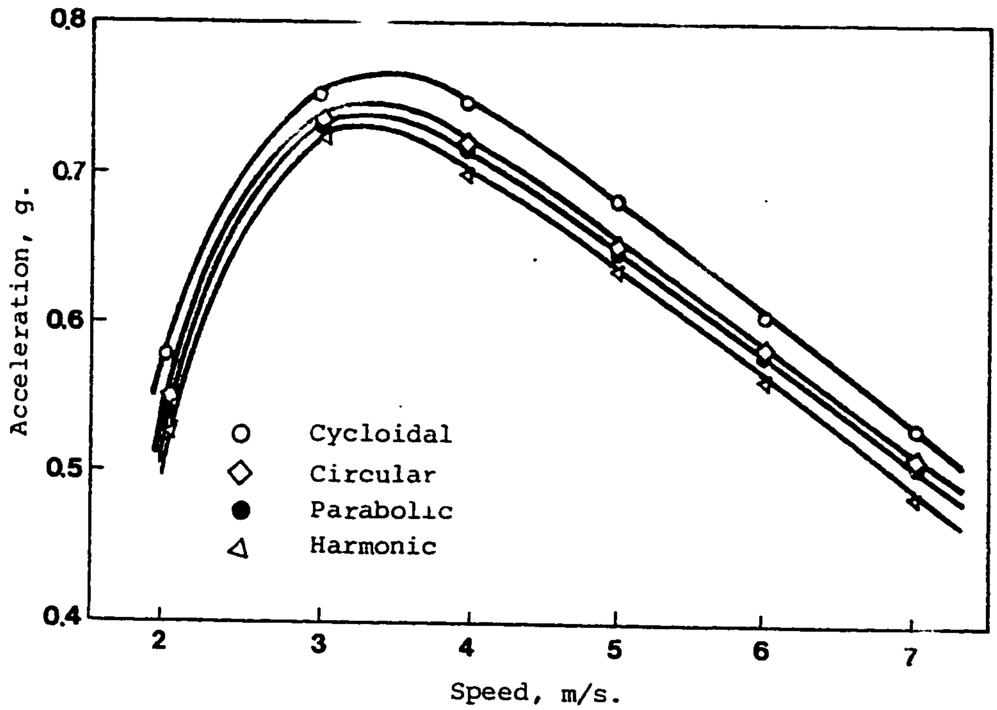


Fig.10 Effect of crossing speed on the acceleration of the driver.

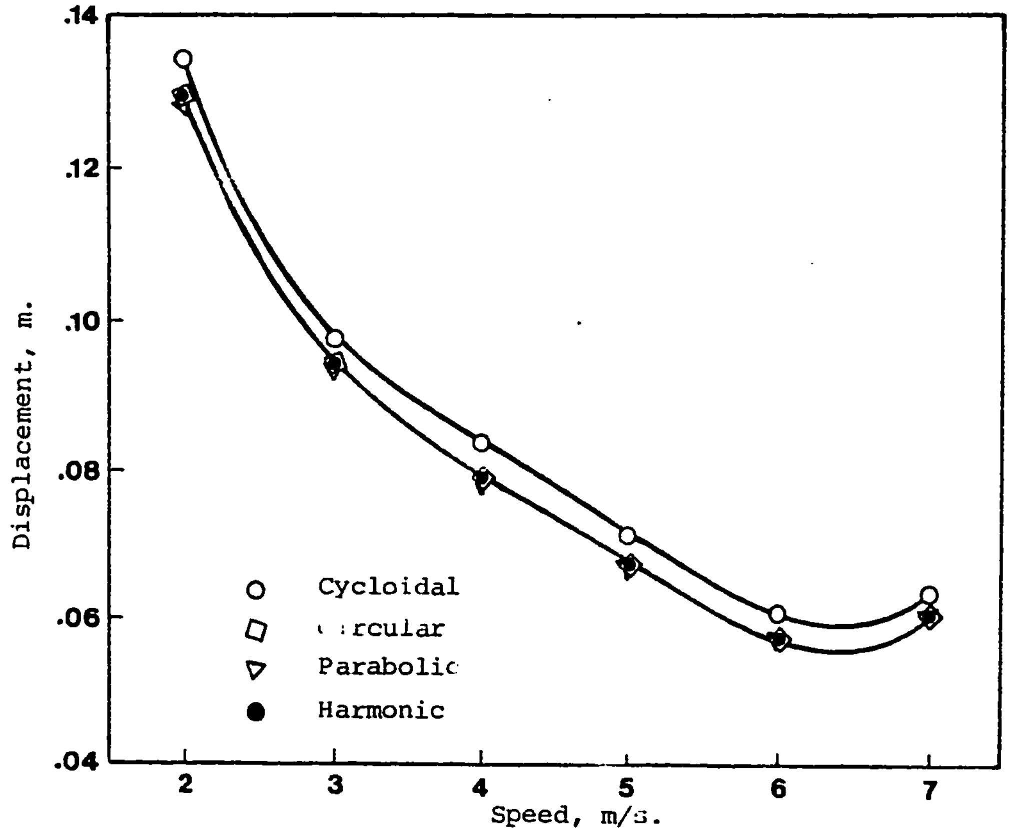


Fig.11 Effect of crossing speed on the vertical displacement of the vehicle's center of mass.

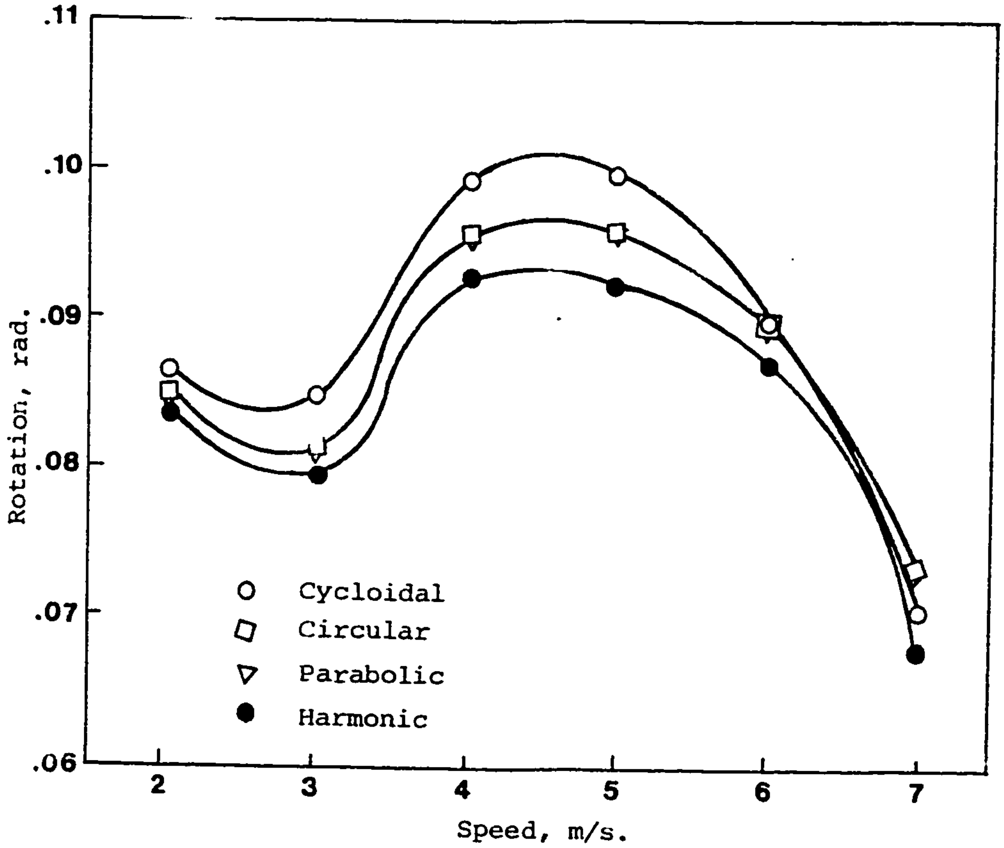


Fig.12 Effect of crossing speed on the pitching of the vehicle.

parameters are choosen to reflect those dynamic effects. These are the driver's acceleration, and the vehicle heave and pitching motions. Fig.10 shows the relation between the maximum acceleration of the driver and the speed of the vehicle as it cross the different humps. It can be seen that the maximum effect takes place at about 3.5 m/s (12.6 Km/hr). The effect of the different profiles is very much the same.

Figs.11 and 12 show the heaving and pitching motions of the vehicle as it crosses the humps with different speeds. Again in this case, the effect of the four different profiles is very much alike.

Based on this preliminary study a hump profile is selected to act as the "basic" hump for the rest of the study. The "harmonic" hump is selected for that purpose. The reason being its relatively low dynamic effects as it can be seen from Figs.10, 11, and 12. The hump basic width is then 1.5 m, while its basic height is 0.15 m. The vehicle "basic speed" is selected to be 5 m/s (18 Km/hr).

CHAPTER 4

CASE STUDY

4.1 INTRODUCTION

In this chapter the effects of the different hump parameters and vehicle speed are to be throughly investigated. Eight cases have been studied. These are:

1 Effect of smoothing the inlet and the outlet of the hump.

2 Effect of varying the width of the hump.

3 Effect of varying the maximum height of the hump.

4 Effect of adding a flat top to the hump (wide hump).

5 Effect of varying the vehicle crossing speed.

6 Effect of adding a deceleration to the vehicle crossing speed.

7 Effect of using double humps.

8 Effect of using triple humps.

In each of the above eight cases, the parameters of the basic harmonic hump discussed in Chapter (3) are kept fixed. Only the parameters under consideration were given different values. That approach is used to isolate the distinguished effect of the specific parameter and to focus on it.

In each of the eight cases, the following plots are produced:

1 Vertical acceleration of the driver versus time.

2 Vertical displacement of the driver versus time.

3 Vertical motion (heave) of the vehicle center of mass.

4 Vertical displacement of the front wheels.

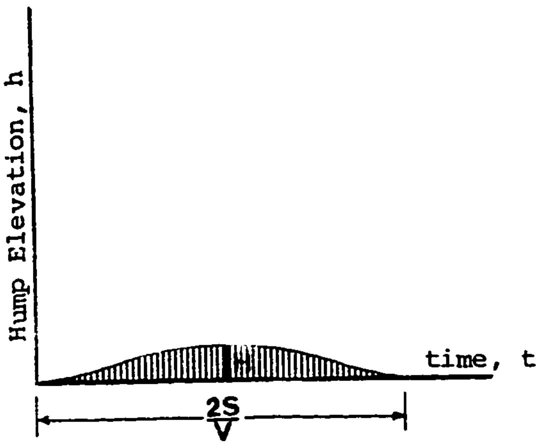
5 Vertical displacement of the rear wheels.

6 Vehicle rotation (pitching) about its center of mass.

Although these are the output data that are chosen to be plotted, the program is able to calculate many other variables (such as vehicle vertical and angular accelerations).

4.2 CASE (1): EFFECT OF SMOOTHING THE INLET AND OUTLET OF THE HUMP.

A smooth inlet/outlet hump, is the one which has zero sloping at its beginning (or entrance) and at its end (or exit). The 'basic' harmonic hump after modifying it by adding smooth inlets and outlets is shown in Fig.13. This case compares the basic harmonic hump with a similar one but after smoothing it. The reflection of that smoothing on the different vehicle and driver dynamic measures is shown in Figs.14 through 19. It can be seen that in general, smoothing the hump reduces the peak values of the dynamic effects by 15-20%. The vertical motion of the front and rear wheels have not improved much because of the smoothing, process. It is also clear that a smooth hump does not change the general pattern of the response.



$$h = \frac{H}{2}\left[1 - \mathrm{Cos}\,\frac{(\pi.V.t)}{S}\right] \qquad 0 \leqslant t \leqslant \frac{S}{V}$$

$$h = \frac{H}{2}\left[1 + \mathrm{Cos}\left\{\frac{\pi.V.(t - S/V)}{S}\right\}\right] \qquad \frac{S}{V} \leqslant t \leqslant \frac{2S}{V}$$

where V = Vehicle speed

Fig.13 Harmonic hump with smooth inlet and outlet

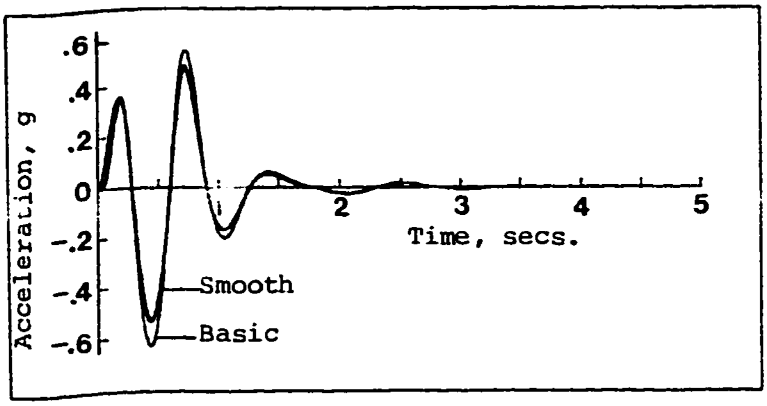


Fig.14 Effect of smoothing the hump on the acceleration of the driver.

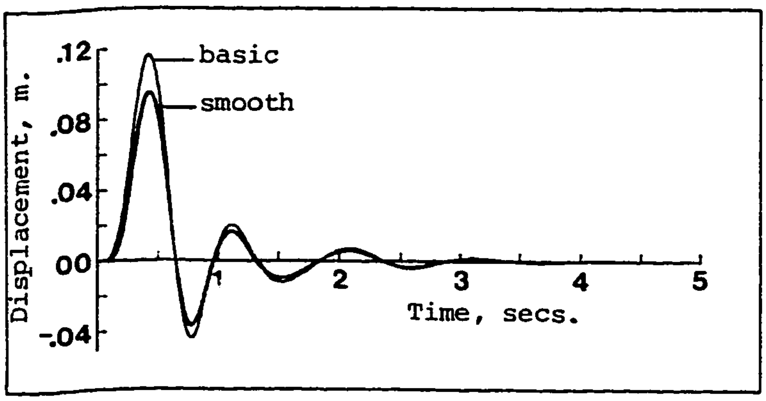


Fig.15 Effect of smoothing the hump on the displacement of the driver.

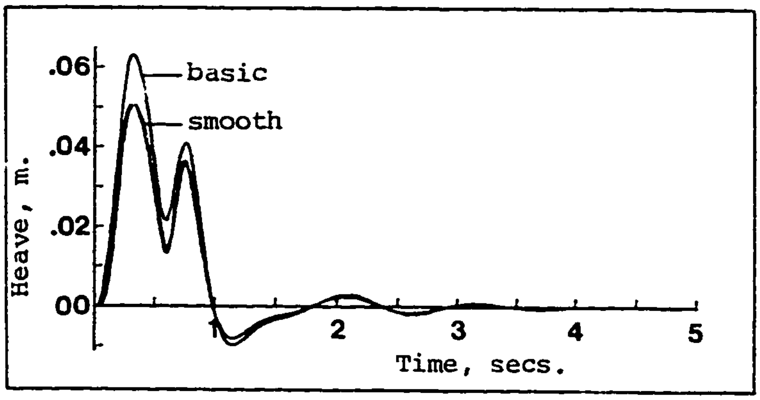


Fig.16 Effect of smoothing the hump on the heaving of the vehicle.

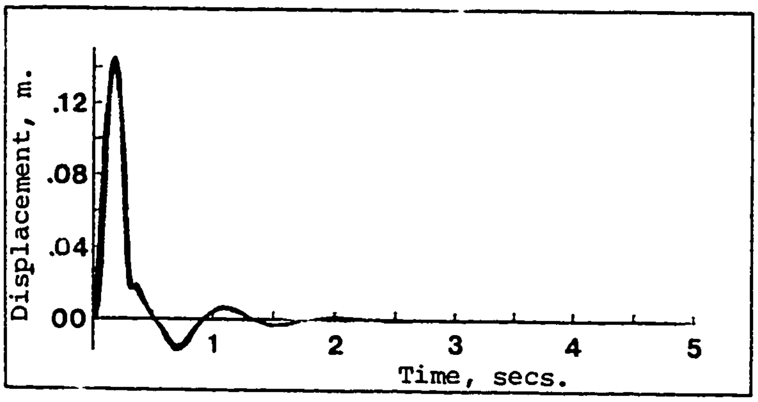


Fig.17 Effect of smoothing the hump on the motion of the front wheels.

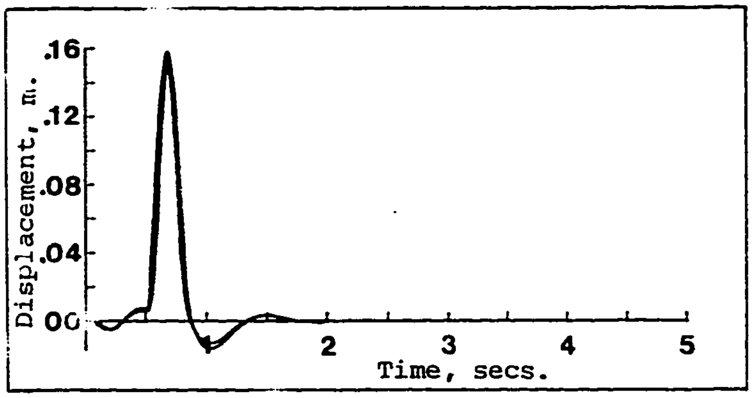


Fig.18 Effect of smoothing the hump on the motion of the rear wheels.

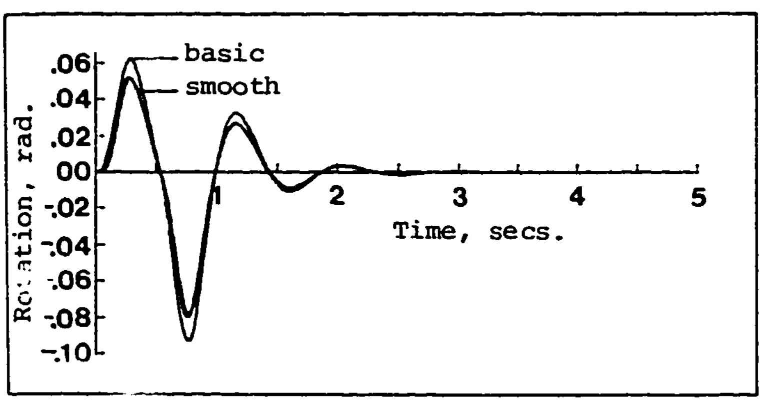


Fig.19 Effect of smoothing the hump on the pitching of the vehicle.

4.3 CASE (II): EFFECT OF VARYING THE WIDTH OF THE HUMP

Four different hump widths, other than the basic width (1.5 m), are used. These are: 1.00, 1.25, 1.75, and 2.00 m. Figs.20 through 25 show the effects of varying the width of the hump on the various vehicle and driver dynamic measures. A general conclusion can be drawn upon examining the response figures: as the hump grows wider, the dynamic effects are more pronounced and, of course, the response time is longer.

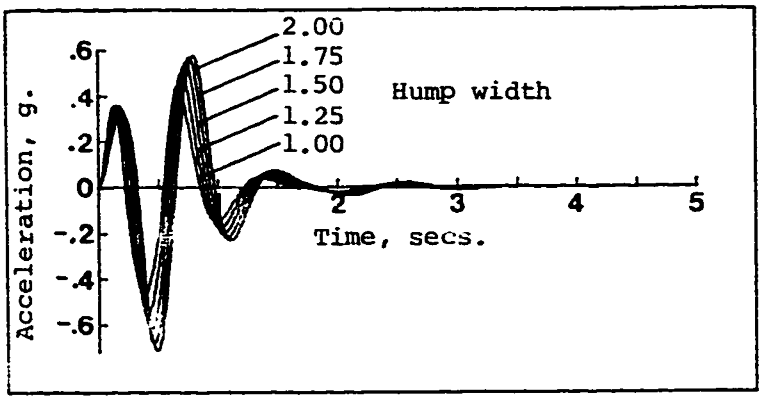


Fig.20 Effect of increasing the width of the hump on the acceleration of the driver.

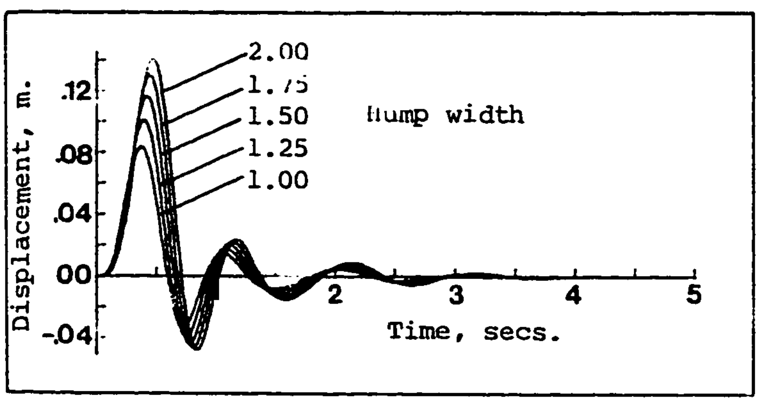


Fig.21 Effect of increasing the width of the hump on the displacement of the driver.

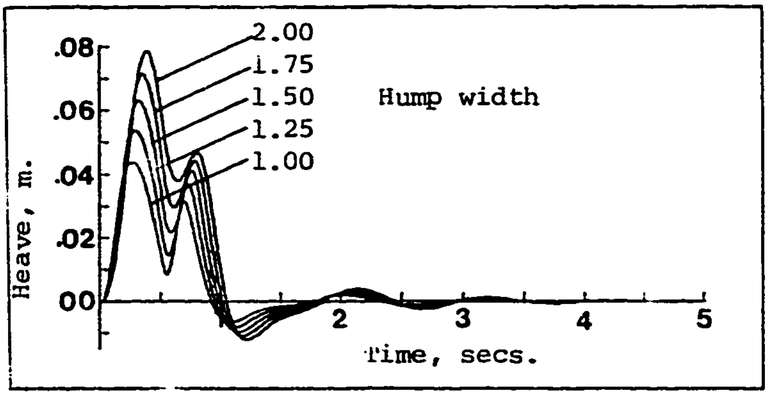


Fig.22 Effect of increasing the width of the hump on the heaving of the vehicle.

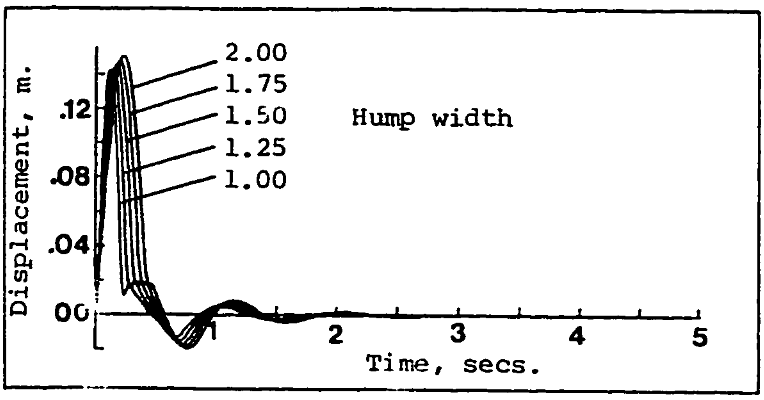


Fig.23 Effect of increasing the width of the hump on the motion of the front wheels.

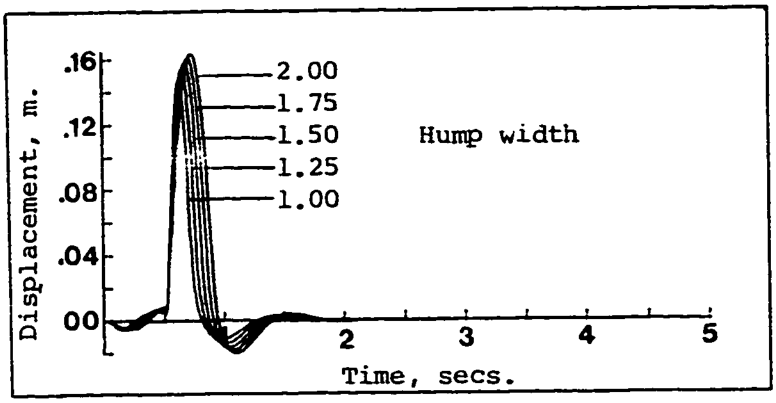


Fig.24 Effect of increasing the width of the hump on the motion of the rear wheels.

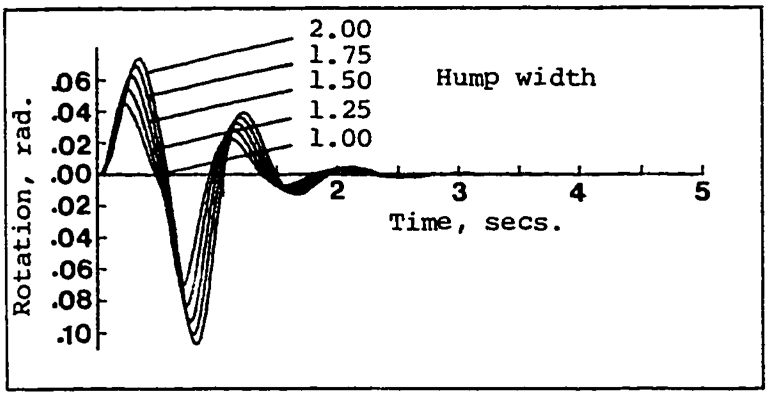


Fig.25 Effect of increasing the width of the hump on the pitching of the vehicle.

4.4 CASE (III): EFFECT OF VARYING THE MAXIMUM HEIGHT OF THE HUMP

Five different maximum hump heights are used. They are: 0.09, 0.11, 0.13, 0.15 (basic), and 0.17 m. The effect of varying the maximum height of the hump is shown in Figs.26 through 31. As in the previous case, the effect is very linear. The higher the hump, the more severe its dynamic effect is.

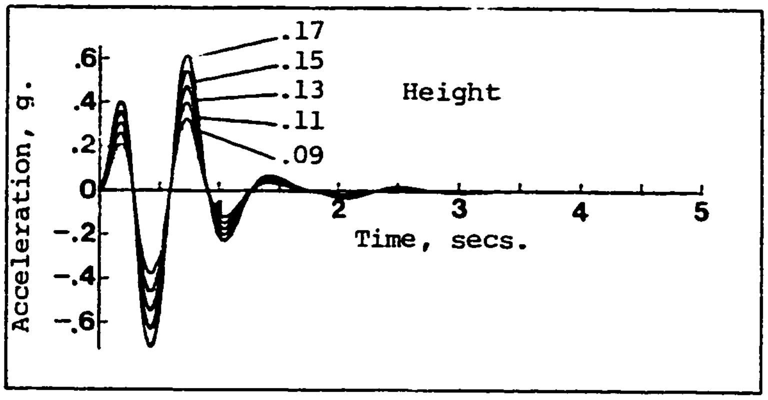


Fig.26 Effect of increasing the height of the hump on the acceleration of the driver.

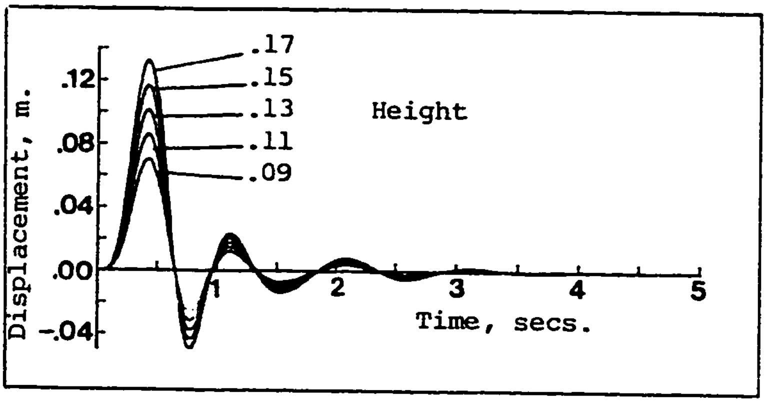


Fig.27 Effect of increasing the height of the hump on the displacement of the driver.

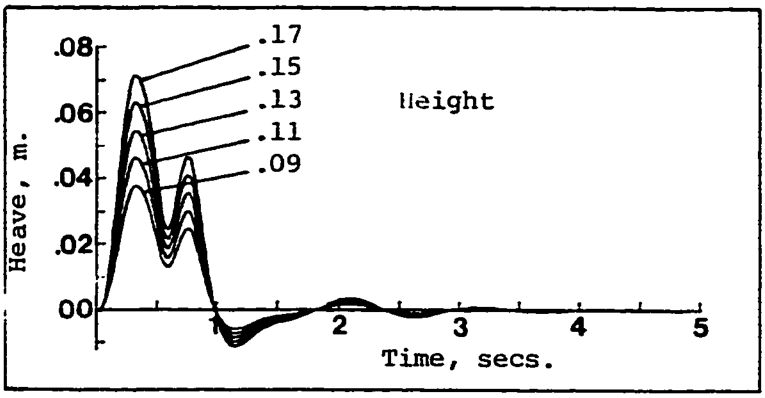


Fig.28 Effect of increasing the height of the hump on the heaving of the vehicle.

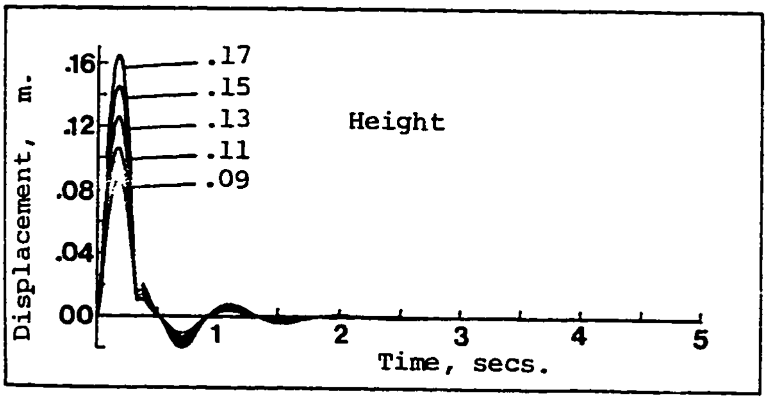


Fig.29 Effect of increasing the height of the hump on the motion of the front wheels.

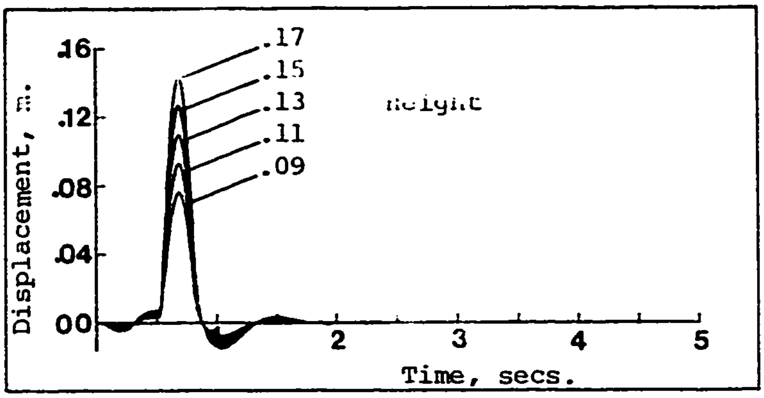


Fig.30 Effect of increasing the height of the hump on the motion of the rear wheels.

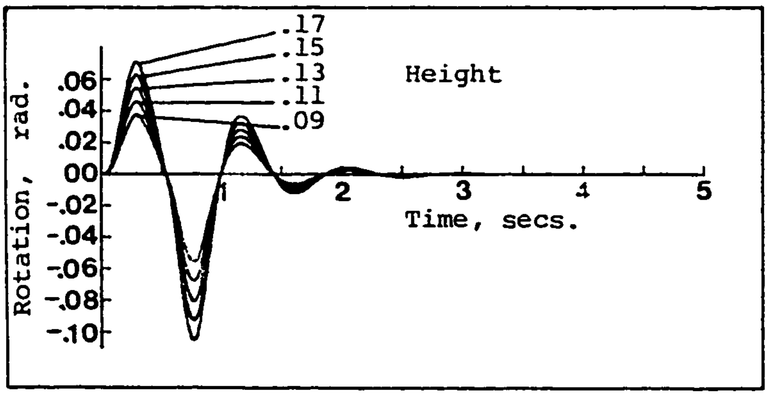


Fig.31 Effect of increasing the height of the hump on the pitching of the vehicle.

4.5 CASE (IV): EFFECT OF ADDING A FLAT TOP TO THE HUMP

A wide hump is created by adding a middle flat top to the basic harmonic hump. The flat top widths used are 0.0, 0.5, 1.0, 1.5, 2.0, and 2.5 m. This makes the total width of the hump equal to 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 m respectively.

ACCELERATION OF THE DRIVER'S SEAT (Fig.32)

The acceleration of the driver's seat is proportional to the force that the driver experiences while crossing the hump. A hump with a total width of 2.0 m generates the highest acceleration. Humps of shorter or longer widths, produce lower magnitudes of accelerations.

DISPLACEMENT OF THE DRIVER'S SEAT (Fig.33)

Within the hump widths used, it can be seen that the maximum displacement of the driver's seat increases gradually as the hump grows wider, up to hump width of 3.0 m. Humps of greater width do not cause any extra increase in the maximum displacement of the driver's seat.

HEAVE MOTION OF THE VEHICLE CENTER OF MASS (Fig.34)

The vehicle wheel base is 2.5 m. The dynamic response in this case is very much related to the ratio between the width of the flat top and the wheel base. For the shortest hump (one of zero flat top), the two peaks reflect the instants when the front and rear wheels ride the top of the hump. These two peaks gradually smoothen as the hump width is increased. Finally, as the width of the flat top reaches that of the wheel base, the two peaks merge into a single peak of height close to that of the hump. Any variation in that peak is attributed to the oscillation of the suspension system. Extrapolating on the above results, one can predict that the maximum displacement of the vehicle center of mass is not expected to be more than that caused by a hump with a total width of 4.0 m.

DISPLACEMENT OF FRONT AND REAR WHEELS (Figs.35 and 36)

The response of the rear and front wheels can be divided into two distinguished zones: the first is due to the ramp effect of the hump, and the second is due to the dynamics of the suspension system while being on the hump. It can be seen that the second zone increases with the increase in the hump width.

PITCHING MOTION OF THE VEHICLE (Fig.37)

The maximum pitching of the vehicle body reaches its highest value while crossing the hump with total width of 3.0 m. Increasing the width of the hump tends to reduce the maximum values of the pitching angles.

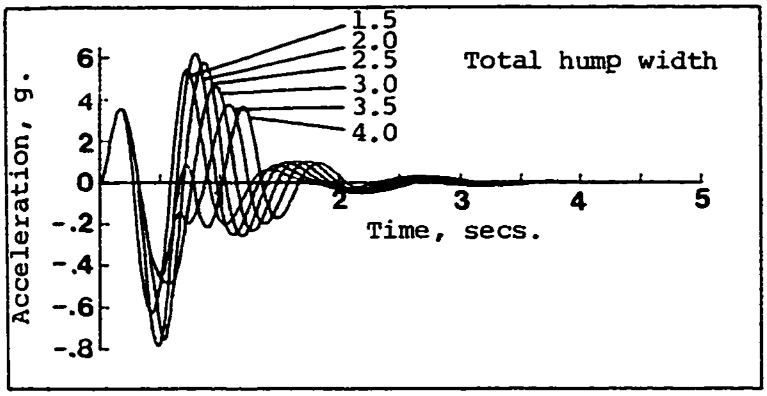


Fig.32 Effect of wide humps on the acceleration of the driver.

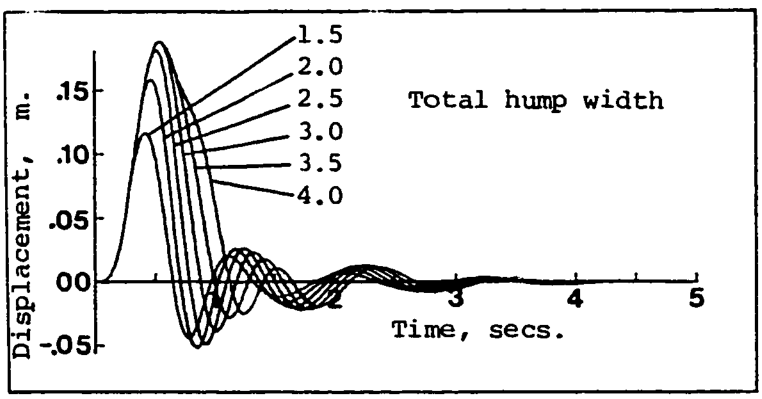


Fig.33 Effect of wide humps on the displacement of the driver.

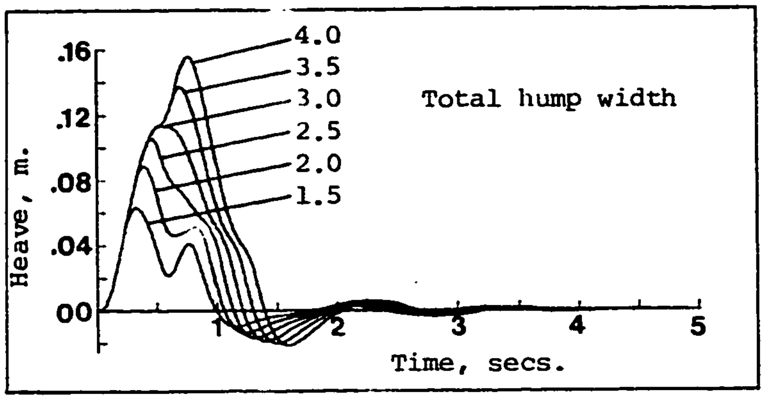


Fig.34 Effect of wide humps on the heaving of the vehicle.

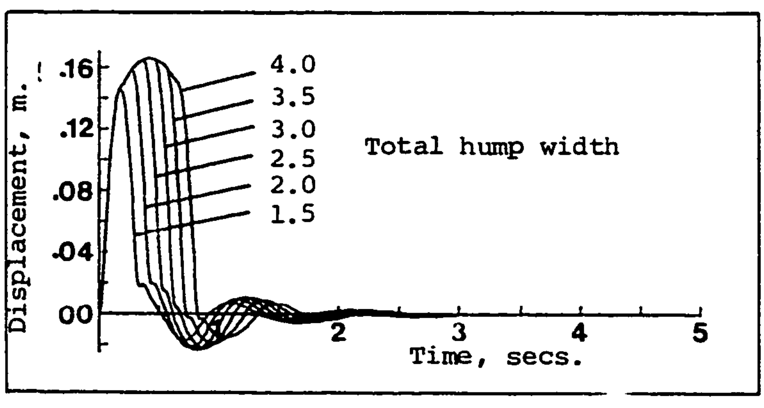


Fig.35 Effect of wide humps on the motion of the front wheels.

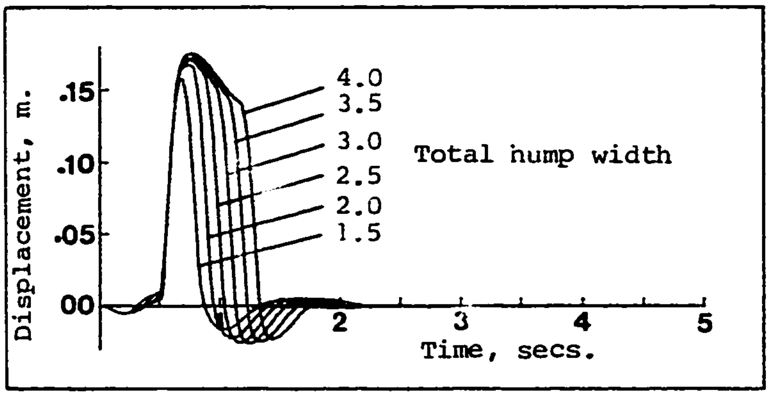


Fig.36 Effect of wide humps on the motion of the rear wheels.

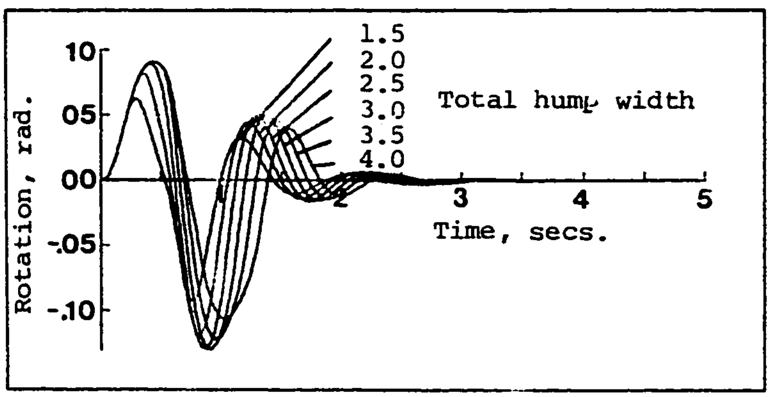


Fig.37 Effect of wide humps on the pitching of the vehicle.

4.6 CASE (V): EFFECT OF VEHICLE CROSSING SPEED

The significance of the high speed is that the vehicle crosses the hump in shorter time. This can be noticed in all of the figures representing this case (Figs.38 through 43). Five different constant crossing speeds are tried. They are 2 m/s (7.2 Km/hr), 3 m/s (10.8 Km/hr), 4 m/s (14.4 Km/hr), 5 m/s (18.0 Km/hr), and 6 m/s (21.6 Km/hr).

In general, it can be seen from Figs.38 and 39 that the highest maximum acceleration that the driver experiences is the one associated with the 3 m/s speed. While the highest maximum displacement of his seat takes places as he crosses the hump with a speed of 4 m/s (14.4 Km/hr). At higher speeds the effect of the hump on the driver tends to decrease. This is within the reasonable and safe speed region.

The effect of increasing the crossing speed on the vehicle is different, however. It seems that, in general, with higher speeds, the suspension system tends to stiffen. This can be clearly noticed in Figs.40 through 43.

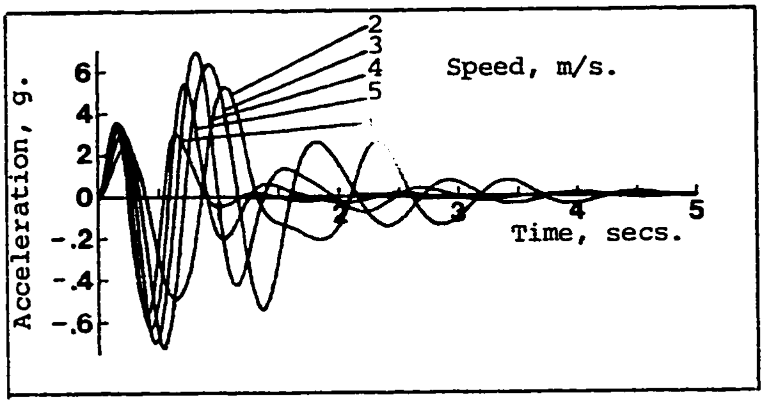


Fig.38 Effect of crossing speed on the acceleration of the driver

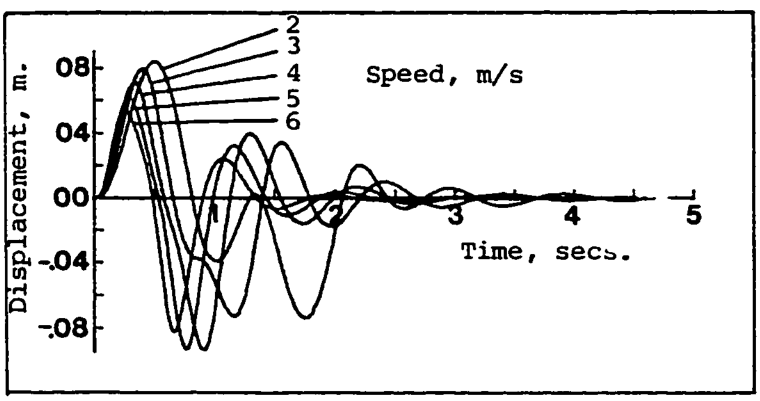


Fig.39 Effect of crossing speed on the displacement of the driver.

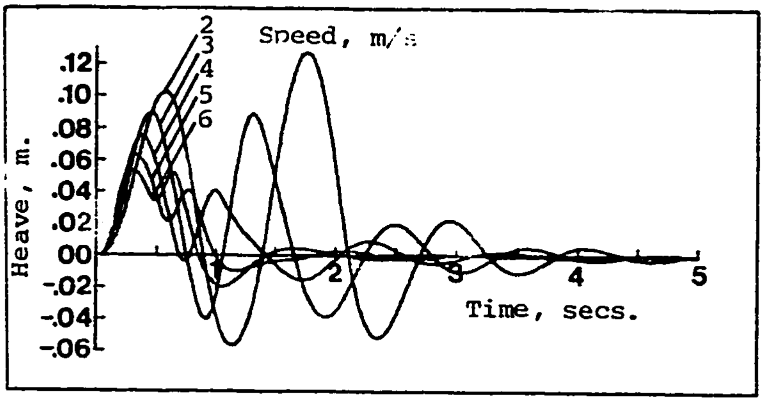


Fig.40 Effect of crossing speed on the heaving of the vehicle.

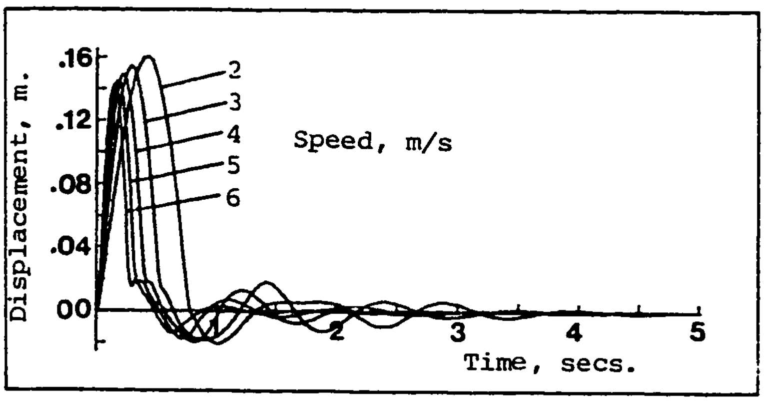


Fig.41 Effect of crossing speed on the motion of the front wheels.

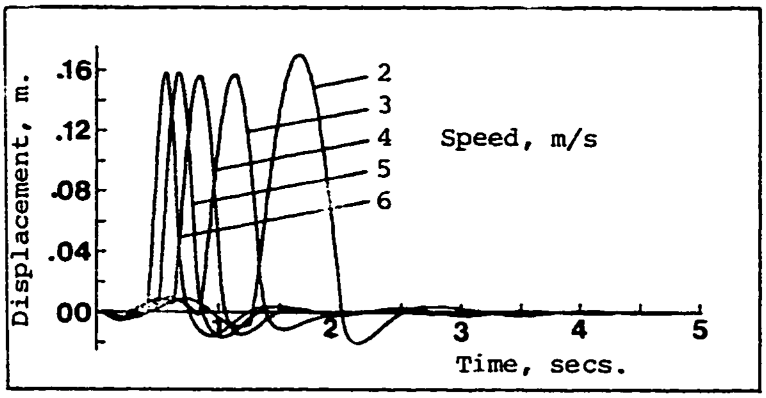


Fig.42 Effect of crossing speed on the motion of the rear wheels.

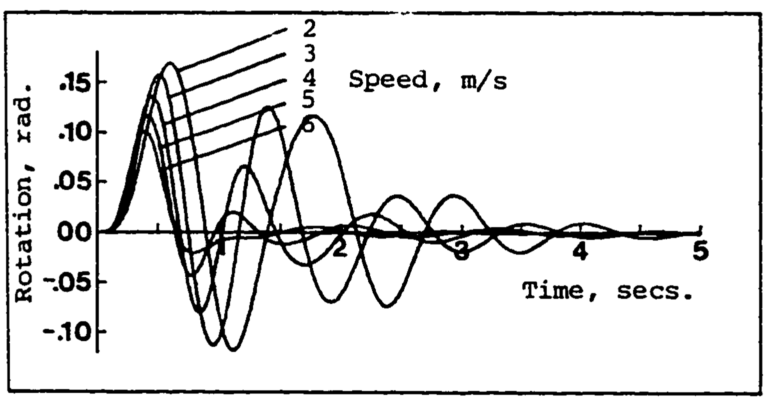


Fig.43 Effect of crossing speed on the pitching of the vehicle.

4.7 CASE (VI): EFFECT OF CORSSING THE HUMP WITH DECELERATION

In all previous cases, vehicle speed, while crossing the hump, was assumed constant. In the case under consideration, the effect of a vehicle crossing the hump with constant deceleration is examined. Three different decelerations are tried: 3.65, 6.25 and 7.41 m/s^2. The vehicle and driver responses are shown in Figs.44 through 49. The response of the vehicle with zero deceleration, i.e., constant crossing speed, is included for reference.

Examining vehicle and driver responses, reveals the fact that the higher the deceleration, the less is the dynamic effect. This is no surprise since higher deceleration means lower crossing speeds. From the practical viewpoint, deceleration can be as high as it is comfortable to the driver and safe to the vehicle.

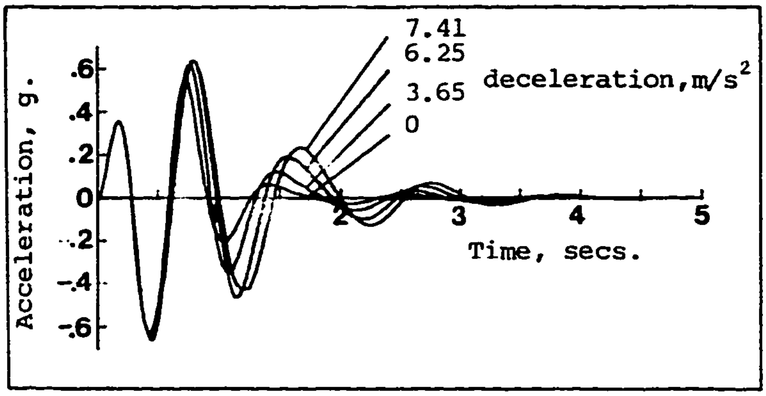


Fig.44 Effect of deceleration on the acceleration of the driver.

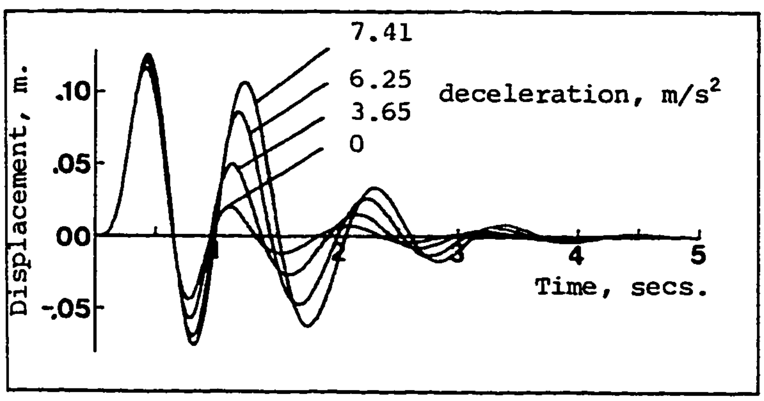


Fig.45 Effect of deceleration in the displacement of the driver.

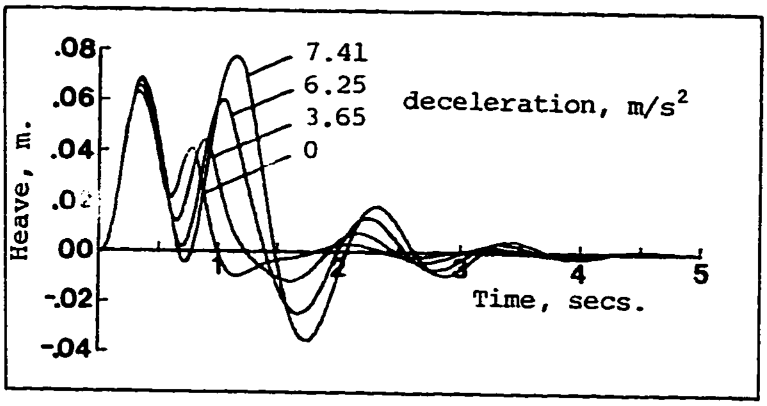


Fig.46 Effect of deceleration on the heaving of the vehicle.

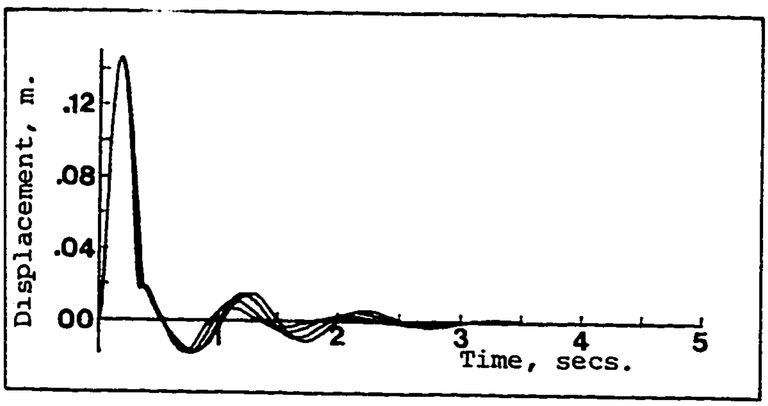


Fig.47 Effect of deceleration on the motion of the front wheels.

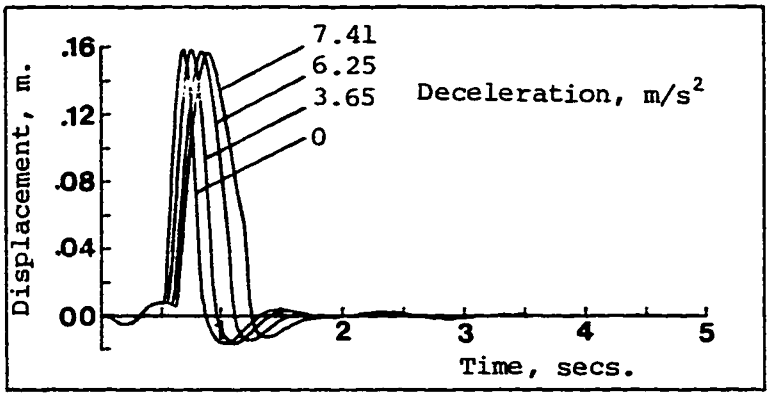


Fig.48 Effect of deceleration on the motion of the rear wheels.

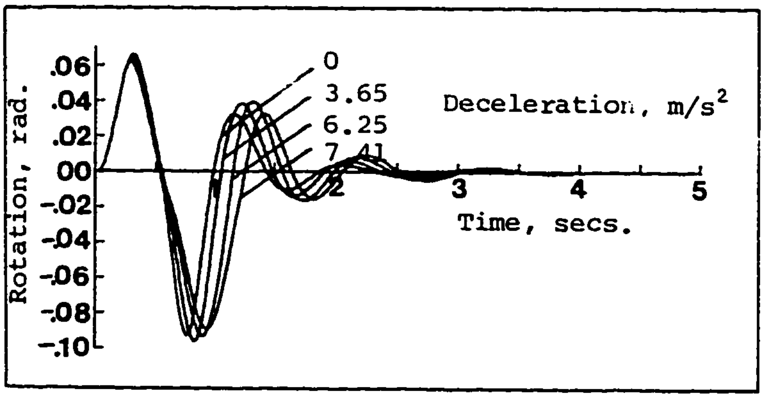


Fig.49 Effect of deceleration on the pitching of the vehicle.

4.8 CASE (VII): DOUBLE HUMP

In this case, the effect of using two consecutive humps is investigated. The objective is to study the effect of varying the spacing between two "basic" humps. Five different spaces are tried: 0.5, 1.0, 2.0, 2.5, and 3.0 m.

From Fig.50, it can be seen that the double hump with spacing of 1.0 m generates the highest positive driver's acceleration. However, if we neglect this isolated effect, and consider the overall picture, then the spacing of 2.0 m produces the highest driver's acceleration. This argument is also true for the vertical displacement of the driver, Fig.51.

The heave motion of vehicle chasis, Fig.52, decreases sharply as the spacing between the humps increased. The vertical motion of the front and rear axles, Figs.53 and 54, is very little affected by the spacing of the humps. Nevertheless, the pitching of the chassis, Fig.55, is very sensitive to that spacing. It offers adverse effect, however, as compared to the heaving motion, Fig.52.

If one is to select a hump spacing for the given vehicle, speed, and hump parameters, it would be of 1.5 m width. It seems that such width gives an overall satisfactory results as far as both vehicle and driver dynamics are concerned. It is worth noting that the basic hump width is also 1.5 m.

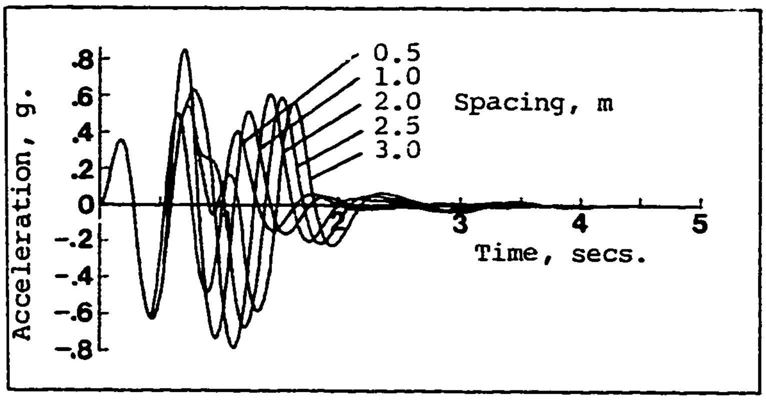


Fig.50 Effect of the double hump on the acceleration of the driver.

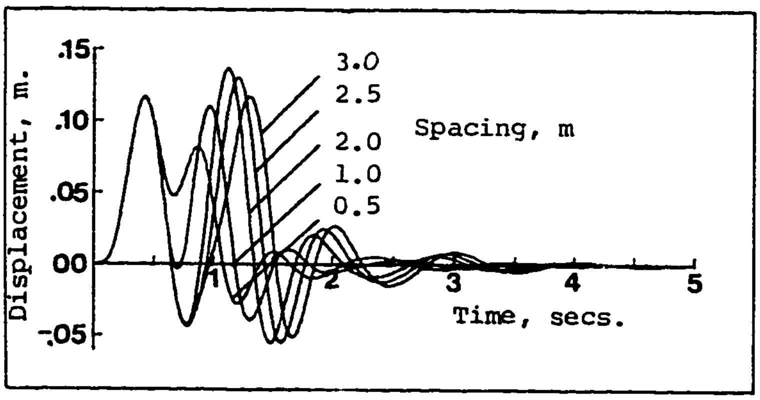


Fig.51 Effect of the double hump on the displacement of the driver.

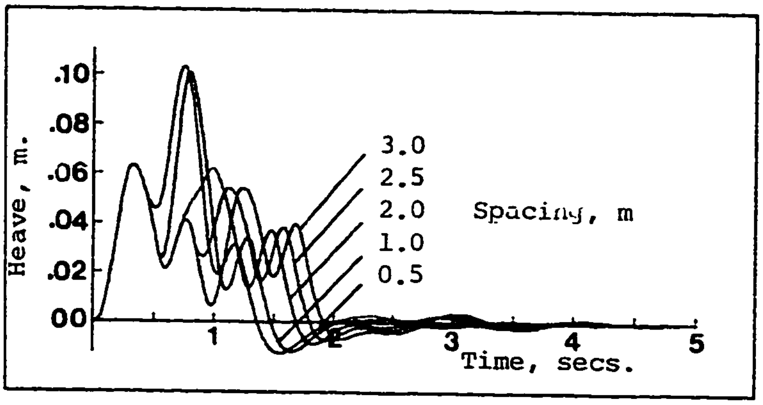


Fig.52 Effect of the double humps on the heaving of the vehicle.

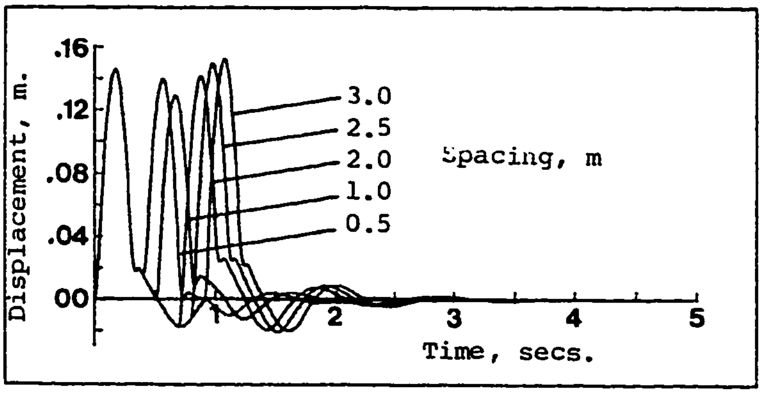


Fig.53 Effect of the double hump on the motion of the front wheels.

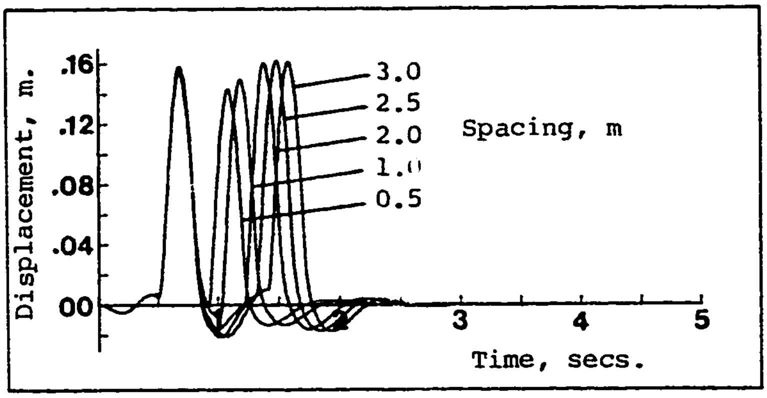


Fig.54 Effect of the double hump on the motion of the rear wheels.

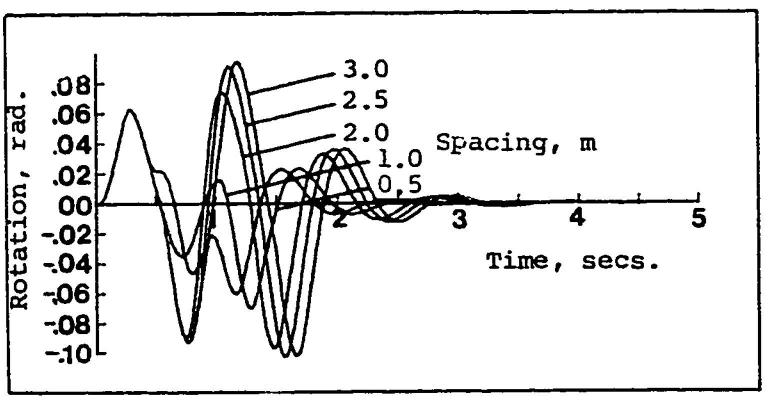


Fig.55 Effect of the double hump on the pitching of the vehicle.

4.9 CASE (VIII): TRIPLE HUMP

In this case three consecutive humps with equal spacings are tried. The spacing used is 1.0, 2.0, and 3.0 m. In general, as can be seen from the response of both vehicle and driver, Fig.56 through 61, the addition of the third hump does not add any drastic change to the dynamic response. The most significant effect is the fact that addition of a third hump causes the suffering of both driver and vehicle to last longer. With three consecutive humps some parameters such as vehicle speed and wheel base become of greater influence on the dynamic response. It is very difficult to come up with a rational design of triple humps that can be safely used with a wide spectrum of vehicles with various crossing speeds. If it is to be used at all, it should be of spacing equal to the hump width. This recommendation is based on extrapolating the results of the double humps.

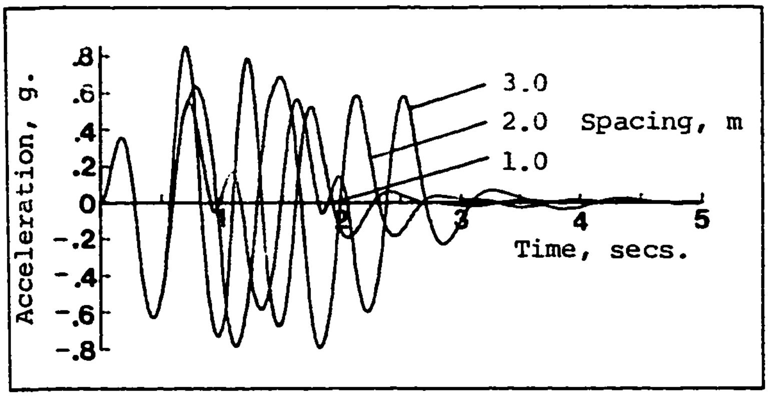


Fig.56 Effect of the triple hump on the acceleration of the driver.

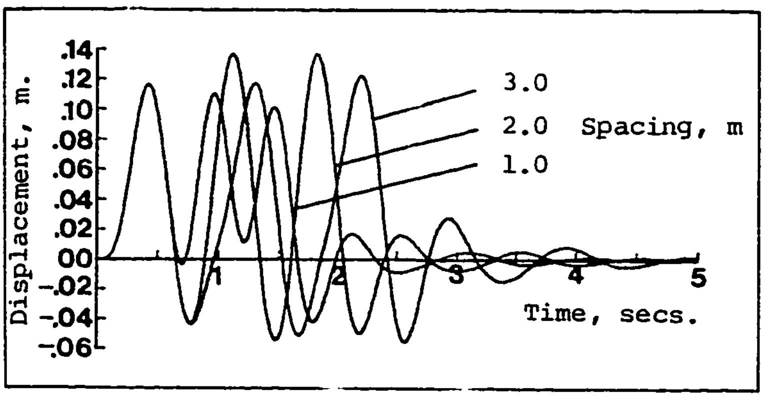


Fig.57 Effect of the triple hump on the displacement of the driver.

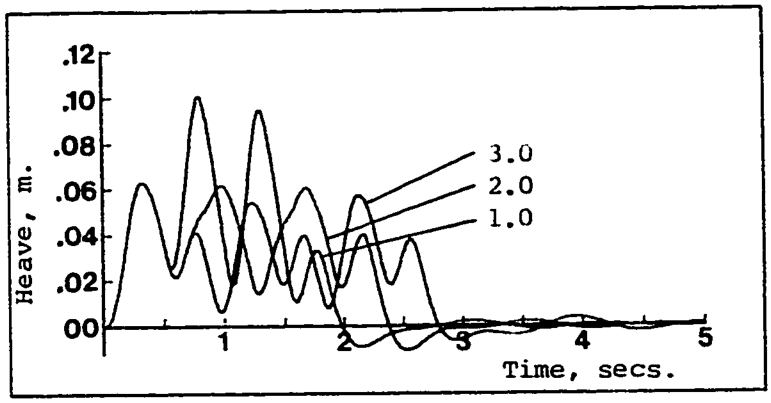


Fig.58 Effect of the triple hump on the heaving of the vehicle.

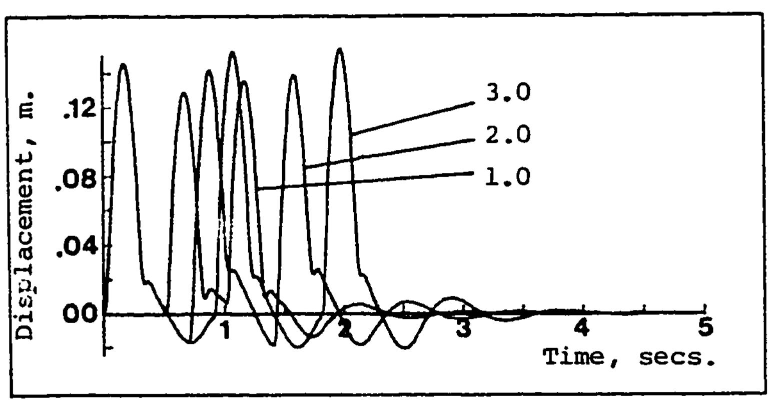


Fig.59 Effect of the triple hump on the motion of the front wheels.

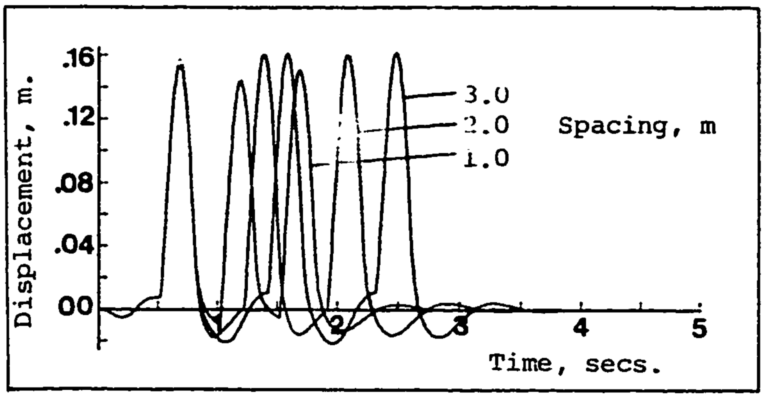


Fig.60 Effect of the triple hump on the motion of the rear wheels.

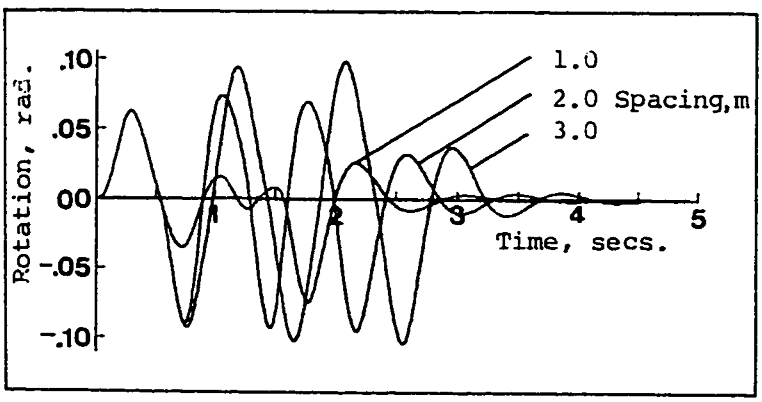


Fig.61 Effect of the triple hump on the pitching of the vehicle.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

A mathematical model has been developed to study the dynamics of both vehicle and driver, while crossing speed control humps. Based on the analysis of the extensive computer-study, the following conclusions and recommendations can be specified:

1 Speed control humps are effective in controlling speeding vehicles. They have to be properly designed, well constructed and carefully positioned. Sufficient signs should be posted well ahead of hump locations.

2 The profile of the humps, whether circular, parabolic, cycloidal, or harmonic does not make any significant effect on its dynamic effects on vehicle and driver.

3 The ride comfort is directly proportional to the vertical acceleration of the driver. This acceleration should not exceed 0.6 g.

4 Smoothing the inlet and outlet of a hump is very effective in reducing its dynamic impact. Up to 20% reduction in the dynamic effects can be achieved by using "smooth" humps.

5 The ratio between the width of the hump to the wheel base is an important parameter in evaluating hump dynamics. For short humps ($B > 25$), dynamic effects grow stronger with the width. If a short hump is to be used, it should be of a width of the same magnitude as the wheel base.

6 If a wide hump is to be used, the width of the middle flat portion should be equal to the wheel base.

7 High humps produce more severe dynamic effects. For safety considerations and to protect the bottom of low slung vehicles, the maximum height of the hump should not exceed 8 cm.

8 Very short humps can be crossed at relatively high speeds without much discomfort. For humps of the recommended shape and dimensions, the ideal design speed is about 15 Km/hr.

9 Double hump magnify the dynamic effects. If they are to be used at all, the spacing between them should be of the same magnitude as the width of the hump.

10 Triple humps cannot be designed on rational basis. They are not suitable for use as speed control humps.

REFERENCES

1. Dahlberg, T., *"Parametric Optimization of a 1-DOF Vehicle Travelling on a Randomly Profiled Road"*, J. of Sound and Vibration, 55(2), pp.245-253, 1977.

2. Dahlberg, T., *"An Optimized Speed-Controlled Suspension of a 2-DOF Vehicle Travelling on a Randomly Profiled Road"*, J. of Sound and Vibration, 62(4), pp.541-564, 1979.

3. Dahlberg, T., *"Optimization Criteria for Vehicles Travelling on Randomly Profiled Road - A Survey"*, Vehicle Sys. Dyn. 8, pp. 239-252, 1979.

4. Good, M., *"Prescribed Trajectory Vehicle Model"*, The Dynamic of Vehicle on Road and Tracks, Proc. 6th VSD-2nd IUTAM Symposium, Vienna, 1977, Ed. A. Slibar and H.Springer, Sweis & Zeitlinger Amsterdam, pp.60-84, 1978.

5. Karnopp, D., *"Vehicle Response to Stochastic Roadways"*, Vehicle System Dynamics 7, pp.97-109, 1978.

6. Muller, P., Popp, K., and Schiehlen, W., *"Covariance Analysis of Nonlinear Guideway-Vehicle-Systems"*, Vehicle System Dynamics, 8, pp.171-177, 1979.

7. Rinonapoli, L., and Bergomi, R., *"A 14 Degree of Freedom Mathematical Model to Predict Car Handling Behavior on Smooth and Bumpy Roads"*, The Dynamic of Vehicle on Road and Tracks, Proc. 6th VSD-2nd IUTAM Symposium, Vienna, 1977, Ed. A.Slibar and H. Springer, Sweis & Zeitlinger Amsterdam, pp.86-109, 1978.

8. Sachs, H., *"An Adaptive Control for Vehicle Suspensions"*, Vehicle System Dynamics, 8, pp.201-206, 1979.

9. Schiehlen, W., *"Dynamic Analysis of Suspension Systems"*, Vehicle Sys. Dyn., 6, pp.56-58, 1977.

10. Snyder, J., and Wormley, D., *"Dynamic Interactions Between Vehicles and Elevated Flexible Randomly Irregular Guideways"*, J. of Dynamic Systems, Measurements, and Control, pp.23-33, March 1977.

11. Sumner, R., and Baguley, C., *"Speed Control Humps in Norwich and Haringey"*, Transport and Road Research Laboratory, Crowthorne, Berkshire, Supplementary Report 423, 1978.

12. Sumner, R., Burton, I., and Baguley, C., *"Speed Control Humps in Cuddesdon Way, Cowley, Oxford"*, Transport and Road Research Laboratory, Supplementary Report 350, Crowthorne, Berkshire, 1978.

13. Watts, G., *"Road Humps for the Control of Vehicle Speeds"*, Transport and Road Research Laboratory, TRRL Report LR 597, Crowthorne, Berkshire, 1973.

A P P E N D I X "A"

```
//ACONASZL JOB 430-24ME-SENC.,'Y.AL NASSAR',CLASS=E,
//         MSGCLASS=T
//   EXEC  FTG1CL
//FORT.SYSPRINT DD SYSOUT=(A,,0851)
//FORT.SYSIN DD *
C       ..............................................
C       THIS PROGRAM IS TO SOLVE SYSTEM OF DIFFERENTIAL
C       EQUATINS DESCRIBING 5-DOF VEHICLE MODEL CROSSING
C       A SINGLE HUMP(BASIC) WITH CONSTANT SPEED
C       USING RKGS SUBROUTINE
C       ..............................................
        DIMENSION PRMT(7),Y(10),DERY(10),AUX(8,10)
        EXTERNAL FCT ,OUTP
        COMMON/BONE/ IP
        COMMON /BTWO/ HF,DHF,HR,DHR
        COMMON/BTHREE/ AMAX,AMIN,YMAX(10),YMIN(10)
        PRMT(1)= 0.0
        PRMT(2)= 5.0
        PRMT(3)=0.01
        PRMT(4)=0.00001
        DO 70 K=1,10
        Y(K)=0.0
        DERY(K)=0.1
   70   CONTINUE
C
C       TO FIND THE MAX. & MIN. OF Y(I),I=1,3,5,7,9
C
        AMAX=0.0
        AMIN=0.0
        DO 3 J=1,10,2
        YMAX(J)=0.0
        YMIN(J)=0.0
    3   CONTINUE
        NDIM= 10
C.....
C       IP=COUNTER
C.....
        IP=0
C.....
C       CALL   RKGS SUBROUTINE
C.....
        CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C.....
C        PRINT OUT THE MAX. & MIN.OF Y(I),I=1,3,5,7,9
C.....
        WRITE(6,300) AMAX,(YMAX(K),K=1,10,2)
        WRITE(6,300) AMIN,(YMIN(K),K=1,10,2)
  300   FORMAT(/,41X,6F10.5)
        STOP
        END
C.....
C       FCT IS A SUBROUTINE CALLED BY RKGS SUB.
C.....
        SUBROUTINE FCT(X,Y,DERY)
        DIMENSION DERY(10),Y(10)
        COMMON /BTWO/ HF,DHF,HR,DHR
        REAL KS,KF,KR,KT,MS,MV,IV,MT,L1,L2
C.....
C       VEHICLE MODEL PARAMETERS
```

```
C.....
C      KS=DRIVER'S SEAT STIFFNESS  (N/M)
C      KF=FRONT SUSPENSION STIFFNESS (N/M)
C      KR=RAER  SUSPENSION STIFFNESS (N/M)
C      KT=TIRE STIFFNESS  (N/M)
C.....
      KS=10000.0
      KF=40000.0
      KR=40000.0
      KT=180000.0
C.....
C      CS=DRIVER'S SEAT DAMPING (N.S/M)
C      CF=FRONT SUSPENSION DAMPING (N.S/M)
C      CR=REAR  SUSPENSION DAMPING (N.S/M)
C      CT=TIRE DAMPING  (N.S/M)
C.....
      CS=500.0
      CF=3000.0
      CR=6000.0
      CT=350.0
C.....
C      MS=MASS OF DRIVER'S SEAT (KG)
C      MV=MASS OF VEHICLE BODY (KG)
C      MT=MASS OF TIRE (KG)
C.....
      MS=100.0
      MV=2000.0
      MT=50.0
C.....
C      IV=PITCH INERTIA OF VEHICLE BODY (ABOUT CG) (KG.M**2)
C.....
      IV=2000.0
C.....
C      A=LENTGH;DRIVER'S SEATTO FRONT SUSPENSION       (M)
C      L1=LENGH;FRONT SUSPENSION TO VEHICLE BODY CG  (M)
C      L2=LENGH;REAR   SUSPENSION TO VEHICLE BODY CG  (M)
C      B=LENGH;FRONT SUSPENSION TO REAR SUSPENSION   (M)
C.....
      A=0.4
      L1=1.3
      L2=1.2
      B=L1+L2
C .....
C          EQUATIONS OF MOTION
C .....
      DERY(1)=Y(2)
      DERY(2)=-1/MS*(KS*(Y(1)-(Y(3)+A*Y(9)))+CS*(Y(2)-(Y(4)+A*Y(10))))
      DERY(3)=Y(4)
      DERY(4)=-1/MV*(KS*(Y(3)+A*Y(9)-Y(1))+CS*(Y(4)+A*Y(10)-Y(2))+KF*
     1(Y(3)+L1*Y(9)-Y(5))+CF*(Y(4)+L1*Y(10)-Y(6))+KR*(Y(3)-L2*Y(9)-
     2Y(7))+CR*(Y(4)-L2*Y(10)-Y(8)))
      DERY(5)=Y(6)
      DERY(6)=-1/MT*(KF*(Y(5)-(Y(3)+L1*Y(9)))+CF*(Y(6)-(Y(4)+L1*Y(10)))
     1+KT*(Y(5)-HF)+CT*(Y(6)-DHF))
      DERY(7)=Y(8)
      DERY(8)=-1/MT*(KR*(Y(7)-(Y(3)-L2*Y(9)))+CR*(Y(8)-(Y(4)-L2*Y(10)))
     1+KT*(Y(7)-HR)+CT*(Y(8)-DHR))
      DERY(9)=Y(10)
      DERY(10)=-1/IV*(KF*L1*(Y(3)+L1*Y(9)-Y(5))+CF*L1*(Y(4)+L1*Y(10)
```

```
     1-Y(6))-KR*L2*(Y(3)-L2*Y(9)-Y(7))-CR*L2*(Y(4)-L2*Y(10)-Y(8))
     2+KS*A*(Y(3)+A*Y(9)-Y(1))+CS*A*(Y(4)+A*Y(10)-Y(2)))
      RETURN
      END
C.....
C     OUTP IS A SUB. CALLED BY A RKGS SUB.
C.....
      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      REAL L1,L2
      DIMENSION Y(10),DERY(10),PRMT(7)
      COMMON /BONE/ IP
      COMMON /BTWO/ HF,DHF,HR,DHR
      COMMON/BTHREE/ AMAX,AMIN,YMAX(10),YMIN(10),SPEED
C.....
C                 HUMP'S PROFILE
C.....
C     ******   HARMONIC   (BASIC) ******
C.....
C     PARA1=EQUATION OF THE FIRST HALF OF THE HUMP AS A FUNCTION
C     OF TIME
C     DPARA1=THE FIRST DEREVETIVE OF PARA1 W.R.T. TIME
C     PARA2=EQUATION OF THE SECOND HALF OF THE HUMP AS A FUNCTION
C     OF TIME
C     DPARA2=THE FIRST DEREVETIVE OF PARA2 W.R.T. TIME
C     X=TIME (S)
C....
      PARA1(X)=HMAX*SIN(PI*SPD*X/(2.*SMAX))
      DPARA1(X)=HMAX*PI*SPD*COS(PI*SPD*X/(2.*SMAX))/(2.*SMAX)
      PARA2(X)=HMAX*COS(PI*SPD*X/(2.*SMAX))
      DPARA2(X)=-HMAX*PI*SPD*SIN(PI*SPD*X/(2.*SMAX))/(2.*SMAX)
C
      PI=3.14159
      L1=1.3
      L2=1.2
      B=L1+L2
C.....
C     SPD=SPEED OF VEHICLE (M/S)
C.....
      SPD=5.0
C.....
C     HUMP'S PARAMETERS
C.....
C     SMAX=WIDTH OF THE HUMP (M)
C     HMAX=MAX. HIGHT OF THE HUMP
C.....
      SMAX=0.75
      HMAX=0.15
      T1=SMAX/SPD
      T2=2*T1
      TT=B/SPD
      T3=TT+T1
      T4=TT+T2
      IF(X.GT.T1) GO TO 10
C.....
C     H=STAND FOR HUMP'S HIGHT    F=STAND FOR FRONT WHEELS
C     D=STAND FOR DEREVETIVE      R=STAND FOR REAR WHEELS
C.....
      HF=PARA1(X)
      DHF=DPARA1(X)
```

```
      HR=0.0
      DHR=0.0
      GO TO 1000
 10   IF(X.GT.T2)  GO TO 20
      HF=PARA2(X-T1)
      DHF=DPARA2(X-T1)
      GO TO 1000
 20   IF(X.GT.TT) GO TO 30
      HF=0.0
      DHF=0.0
      GO TO 1000
 30   IF(X.GT.T3) GO TO 40
      HR=PARA1(X-TT)
      DHR=DPARA1(X-TT)
      GO TO 1000
 40   IF(X.GT.T4) GO TO 50
      HR=PARA2(X-T3)
      DHR=DPARA2(X-T3)
      GO TO 1000
 50   HR=0.0
      DHR=0.0
1000  CONTINUE
      IF(X.EQ.0.0) GO TO 15
      IF(X.LT.PRMT(6)) GO TO 16
C.....
C           OUTPUT DATA
C.....
C     DERY(2)=VERTICAL ACCE. OF THE DRIVER SEAT
C     Y(1)=VERTICAL MOTION OF THE DRIVER'S SEAT
C     Y(3)=VERTICAL MOTION OF THE CENTER OF MASS OF THE VEHICLE
C     Y(5)=VERTICAL MOTION OF FRONT WHEELS
C     Y(7)=VERTICAL MOTION OF REAR  WHEELS
C     Y(9)=VERTICAL ROTATION ABOUT CENTER OF MASS OF THE VEHICLE
C.....
 15   IP=IP+1
      WRITE(6,60) IP,X,HF,HR,(Y(I),I=1,10,2)
 60   FORMAT(2X,I7,2X,9F10.5)
      IF(DERY(2).GT.AMAX) AMAX=DERY(2)
      IF(DERY(2).LT.AMIN) AMIN=DERY(2)
      DO 700 IL=1,10,2
      IF(Y(IL).GT.YMAX(IL)) YMAX(IL)=Y(IL)
 700  IF(Y(IL).LT.YMIN(IL)) YMIN(IL)=Y(IL)
C ......................................
      PRMT(6)=X+.005
 16   RETURN
      END
C
C     ..............................................................
C
C        SUBROUTINE RKGS
C
C        PURPOSE
C           TO SOLVE A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL
C           EQUATIONS WITH GIVEN INITIAL VALUES.
C
C        USAGE
C           CALL RKGS (PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C           PARAMETERS FCT AND OUTP REQUIRE AN EXTERNAL STATEMENT.
C
```

```
C        DESCRIPTION OF PARAMETERS                                     RKGS
C           PRMT   - AN INPUT AND OUTPUT VECTOR WITH DIMENSION GREATER   RKGS
C                    OR EQUAL TO 5, WHICH SPECIFIES THE PARAMETERS OF    RKGS
C                    THE INTERVAL AND OF ACCURACY AND WHICH SERVES FOR   RKGS
C                    COMMUNICATION BETWEEN OUTPUT SUBROUTINE (FURNISHED  RKGS
C                    BY THE USER) AND SUBROUTINE RKGS. EXCEPT PRMT(5)    RKGS
C                    THE COMPONENTS ARE NOT DESTROYED BY SUBROUTINE      RKGS
C                    RKGS AND THEY ARE                                   RKGS
C           PRMT(1)- LOWER BOUND OF THE INTERVAL (INPUT),                RKGS
C           PRMT(2)- UPPER BOUND OF THE INTERVAL (INPUT),                RKGS
C           PRMT(3)- INITIAL INCREMENT OF THE INDEPENDENT VARIABLE       RKGS
C                    (INPUT),                                            RKGS
C           PRMT(4)- UPPER ERROR BOUND (INPUT). IF ABSOLUTE ERROR IS     RKGS
C                    GREATER THAN PRMT(4), INCREMENT GETS HALVED.        RKGS
C                    IF INCREMENT IS LESS THAN PRMT(3) AND ABSOLUTE      RKGS
C                    ERROR LESS THAN PRMT(4)/50, INCREMENT GETS DOUBLED.RKGS
C                    THE USER MAY CHANGE PRMT(4) BY MEANS OF HIS         RKGS
C                    OUTPUT SUBROUTINE.                                  RKGS
C           PRMT(5)- NO INPUT PARAMETER. SUBROUTINE RKGS INITIALIZES     RKGS
C                    PRMT(5)=0. IF THE USER WANTS TO TERMINATE           RKGS
C                    SUBROUTINE RKGS AT ANY OUTPUT POINT, HE HAS TO      RKGS
C                    CHANGE PRMT(5) TO NON-ZERO BY MEANS OF SUBROUTINE   RKGS
C                    OUTP. FURTHER COMPONENTS OF VECTOR PRMT ARE         RKGS
C                    FEASIBLE IF ITS DIMENSION IS DEFINED GREATER        RKGS
C                    THAN 5. HOWEVER SUBROUTINE RKGS DOES NOT REQUIRE    RKGS
C                    AND CHANGE THEM. NEVERTHELESS THEY MAY BE USEFUL    RKGS
C                    FOR HANDING RESULT VALUES TO THE MAIN PROGRAM       RKGS
C                    (CALLING RKGS) WHICH ARE OBTAINED BY SPECIAL        RKGS
C                    MANIPULATIONS WITH OUTPUT DATA IN SUBROUTINE OUTP.  RKGS
C           Y      - INPUT VECTOR OF INITIAL VALUES.  (DESTROYED)        RKGS
C                    LATERON Y IS THE RESULTING VECTOR OF DEPENDENT      RKGS
C                    VARIABLES COMPUTED AT INTERMEDIATE POINTS X.        RKGS
C           DERY   - INPUT VECTOR OF ERROR WEIGHTS.  (DESTROYED)         RKGS
C                    THE SUM OF ITS COMPONENTS MUST BE EQUAL TO 1.       RKGS
C                    LATERON DERY IS THE VECTOR OF DERIVATIVES, WHICH    RKGS
C                    BELONG TO FUNCTION VALUES Y AT A POINT X.           RKGS
C           NDIM   - AN INPUT VALUE, WHICH SPECIFIES THE NUMBER OF       RKGS
C                    EQUATIONS IN THE SYSTEM.                            RKGS
C           IHLF   - AN OUTPUT VALUE, WHICH SPECIFIES THE NUMBER OF      RKGS
C                    BISECTIONS OF THE INITIAL INCREMENT. IF IHLF GETS   RKGS
C                    GREATER THAN 10, SUBROUTINE RKGS RETURNS WITH       RKGS
C                    ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM. ERROR      RKGS
C                    MESSAGE IHLF=12 OR IHLF=13 APPEARS IN CASE          RKGS
C                    PRMT(3)=0 OR IN CASE SIGN(PRMT(3)).NE.SIGN(PRMT(2)-RKGS
C                    PRMT(1)) RESPECTIVELY.                              RKGS
C           FCT    - THE NAME OF AN EXTERNAL SUBROUTINE USED. THIS       RKGS
C                    SUBROUTINE COMPUTES THE RIGHT HAND SIDES DERY OF    RKGS
C                    THE SYSTEM TO GIVEN VALUES X AND Y. ITS PARAMETER   RKGS
C                    LIST MUST BE X,Y,DERY. SUBROUTINE FCT SHOULD        RKGS
C                    NOT DESTROY X AND Y.                                RKGS
C           OUTP   - THE NAME OF AN EXTERNAL OUTPUT SUBROUTINE USED.     RKGS
C                    ITS PARAMETER LIST MUST BE X,Y,DERY,IHLF,NDIM,PRMT.RKGS
C                    NONE OF THESE PARAMETERS (EXCEPT, IF NECESSARY,     RKGS
C                    PRMT(4),PRMT(5),...) SHOULD BE CHANGED BY           RKGS
C                    SUBROUTINE OUTP. IF PRMT(5) IS CHANGED TO NON-ZERO,RKGS
C                    SUBROUTINE RKGS IS TERMINATED.                      RKGS
C           AUX    - AN AUXILIARY STORAGE ARRAY WITH 8 ROWS AND NDIM     RKGS
C                    COLUMNS.                                            RKGS
C                                                                        RKGS
```

```
C        REMARKS
C           THE PROCEDURE TERMINATES AND RETURNS TO CALLING PROGRAM, IF
C           (1) MORE THAN 10 BISECTIONS OF THE INITIAL INCREMENT ARE
C               NECESSARY TO GET SATISFACTORY ACCURACY (ERROR MESSAGE
C               IHLF=11),
C           (2) INITIAL INCREMENT IS EQUAL TO 0 OR HAS WRONG SIGN
C               (ERROR MESSAGES IHLF=12 OR IHLF=13),
C           (3) THE WHOLE INTEGRATION INTERVAL IS WORKED THROUGH,
C           (4) SUBROUTINE OUTP HAS CHANGED PRMT(5) TO NON-ZERO.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           THE EXTERNAL SUBROUTINES FCT(X,Y,DERY) AND
C           OUTP(X,Y,DERY,IHLF,NDIM,PRMT) MUST BE FURNISHED BY THE USER.
C
C        METHOD
C           EVALUATION IS DONE BY MEANS OF FOURTH ORDER RUNGE-KUTTA
C           FORMULAE IN THE MODIFICATION DUE TO GILL. ACCURACY IS
C           TESTED COMPARING THE RESULTS OF THE PROCEDURE WITH SINGLE
C           AND DOUBLE INCREMENT.
C           SUBROUTINE RKGS AUTOMATICALLY ADJUSTS THE INCREMENT DURING
C           THE WHOLE COMPUTATION BY HALVING OR DOUBLING. IF MORE THAN
C           10 BISECTIONS OF THE INCREMENT ARE NECESSARY TO GET
C           SATISFACTORY ACCURACY, THE SUBROUTINE RETURNS WITH
C           ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM.
C           TO GET FULL FLEXIBILITY IN OUTPUT, AN OUTPUT SUBROUTINE
C           MUST BE FURNISHED BY THE USER.
C           FOR REFERENCE, SEE
C           RALSTON/WILF, MATHEMATICAL METHODS FOR DIGITAL COMPUTERS,
C           WILEY, NEW YORK/LONDON, 1960, PP.110-120.
C
C     ..................................................................
C
      SUBROUTINE NASS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C
C
      DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)
      DO 1 I=1,NDIM
    1 AUX(8,I)=.06666667*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)=0.
      CALL FCT(X,Y,DERY)
C
C     ERROR TEST
      IF(H*(XEND-X))38,37,2
C
C     PREPARATIONS FOR RUNGE-KUTTA METHOD
    2 A(1)=.5
      A(2)=.2928932
      A(3)=1.707107
      A(4)=.1666667
      B(1)=2.
      B(2)=1.
      B(3)=1.
      B(4)=2.
      C(1)=.5
      C(2)=.2928932
      C(3)=1.707107
```

```
      C(4)=.5

      PREPARATIONS OF FIRST RUNGE-KUTTA STEP
      DO 3 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.
    3 AUX(6,I)=0.
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0


      START OF A RUNGE-KUTTA STEP
    4 IF((X+H-XEND)*H)7,6,5
    5 H=XEND-X
    6 IEND=1

      RECORDING OF INITIAL VALUES OF THIS STEP
    7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)
      IF(PRMT(5))40,8,40
    8 ITEST=0
    9 ISTEP=ISTEP+1


      START OF INNERMOST RUNGE-KUTTA LOOP
      J=1
   10 AJ=A(J)
      BJ=B(J)
      CJ=C(J)
      DO 11 I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF(J-4)12,15,15
   12 J=J+1
      IF(J-3)13,14,13
   13 X=X+.5*H
   14 CALL FCT(X,Y,DERY)
      GOTO 10
      END OF INNERMOST RUNGE-KUTTA LOOP


      TEST OF ACCURACY
   15 IF(ITEST)16,16,20

      IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
   16 DO 17 I=1,NDIM
   17 AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
   18 IHLF=IHLF+1
      X=X-H
      H=.5*H
      DO 19 I=1,NDIM
```

```
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
   19 AUX(6,I)=AUX(3,I)
      GOTO 9
C
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
   20 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
   21 CALL FCT(X,Y,DERY)
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
   22 AUX(7,I)=DERY(I)
      GOTO 9
C
C     COMPUTATION OF TEST VALUE DELT
   23 DELT=0.
      DO 24 I=1,NDIM
   24 DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF(DELT-PRMT(4))28,28,25
C
C     ERROR IS TOO GREAT
   25 IF(IHLF-10)26,36,36
   26 DO 27 I=1,NDIM
   27 AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GOTO 18
C
C     RESULT VALUES ARE GOOD
   28 CALL FCT(X,Y,DERY)
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
   29 DERY(I)=AUX(7,I)
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)
      IF(PRMT(5))40,30,40
   30 DO 31 I=1,NDIM
      Y(I)=AUX(1,I)
   31 DERY(I)=AUX(2,I)
      IREC=IHLF
      IF(IEND)32,32,39
C
C     INCREMENT GETS DOUBLED
   32 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      IF(IHLF)4,33,33
   33 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)4,34,4
   34 IF(DELT-.02*PRMT(4))35,35,4
   35 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      GOTO 4
C
C
```

```
C        RETURNS TO CALLING PROGRAM
   36 IHLF=11
      CALL FCT(X,Y,DERY)
      GOTO 39
   37 IHLF=12
      GOTO 39
   38 IHLF=13
   39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
   40 RETURN
      END
//LKED.SYSPRINT DD
/*
```